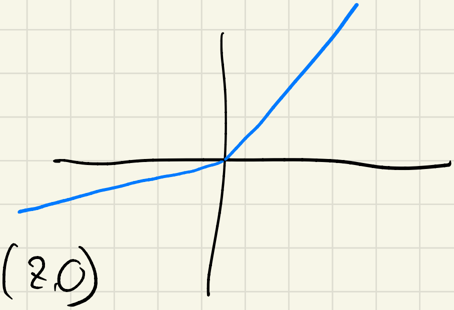
25.3.2020

Leaky - ReLu:



$$\varphi(z) = \max(z,0) + 0.1 \cdot \min(z,0)$$

$$\varphi'(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0.1 & \text{if } z < 0 \end{cases}$$

Jacobians and Backpropagation

Sps: $f: \mathbb{R}^n \to \mathbb{R}^m$

$$\frac{d}{dx} f(x) = \underbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}}_{\in \mathbb{R}^{m \times n}} =: J_f(x) = \frac{\partial f}{\partial x}$$

Taylor series $\to$ linearize f

$$f(x) = f(x_0) + J_f(x_0) \cdot (x - x_0) + o(\|x - x_0\|)$$

Chain rule

$$\frac{d}{dx} g(f(x)) = J_{(g \circ f)}(x) = J_g(f(x)) \cdot J_f(x)$$

Ex: $f(x) = Wx$, $W \in \mathbb{R}^{m \times n}$

$J_f(x) = W$

$\varphi: \mathbb{R} \to \mathbb{R}$

Ex: $f(x) = [\varphi(x_1), \dots \varphi(x_n)]^T$

$$J_f(x) = \begin{pmatrix} \varphi'(x_1) & & 0 \\ & \ddots & \\ 0 & & \varphi'(x_n) \end{pmatrix} = \operatorname{diag}(\varphi'(x))$$

$\varphi(x) = [\varphi(x_1) \dots \varphi(x_n)]$

$\varphi'(x) = [\varphi'(x_1) \dots \varphi'(x_n)]$

Ex: $f(x) = \varphi(Wx)$

$$J_f(x) = J_\varphi(\underbrace{Wx}_{z}) \cdot J_W(x) = \operatorname{diag}(\varphi'(\underbrace{Wx}_{z})) \cdot W$$

$$L = \ell(y; f(W,x)) = \ell_y(f(W,x))$$

$$W = [W^{(1)}, \dots, W^{(L)}] \in \mathbb{R}^{d \cdot n_1 + n_1 \cdot n_2 + \dots + n_L \cdot n_0}$$

$$f = V^{(L)} v^{(L-1)}$$

$$\frac{\partial L}{\partial W^{(L)}} = \underbrace{\frac{\partial L}{\partial f}}_{\delta^{(L)}} \underbrace{\frac{\partial f}{\partial W^{(L)}}}_{v^{(L-1)}}$$

$$\frac{\partial L}{\partial W^{(L-1)}} = \underbrace{\overbrace{\frac{\partial L}{\partial f}}^{\delta^{(L)}} \frac{\partial f}{\partial z^{(L-1)}}}_{\delta^{(L-1)}} \underbrace{\frac{\partial z^{(L-1)}}{\partial W^{(L-1)}}}_{v^{(L-2)}}$$

$$\vdots$$

$$\frac{\partial L}{\partial W^{(i)}} = \underbrace{\overbrace{\frac{\partial L}{\partial f} \frac{\partial f}{\partial z^{(L-1)}} \frac{\partial z^{(L-1)}}{\partial z^{(L-2)}} \cdots}^{\delta^{(i+1)}} \frac{\partial z^{(i+1)}}{\partial z^{(i)}}}_{\delta^{(i)}} \underbrace{\frac{\partial z^{(i)}}{\partial W^{(i)}}}_{v^{(i-1)}}$$