ETH zürich

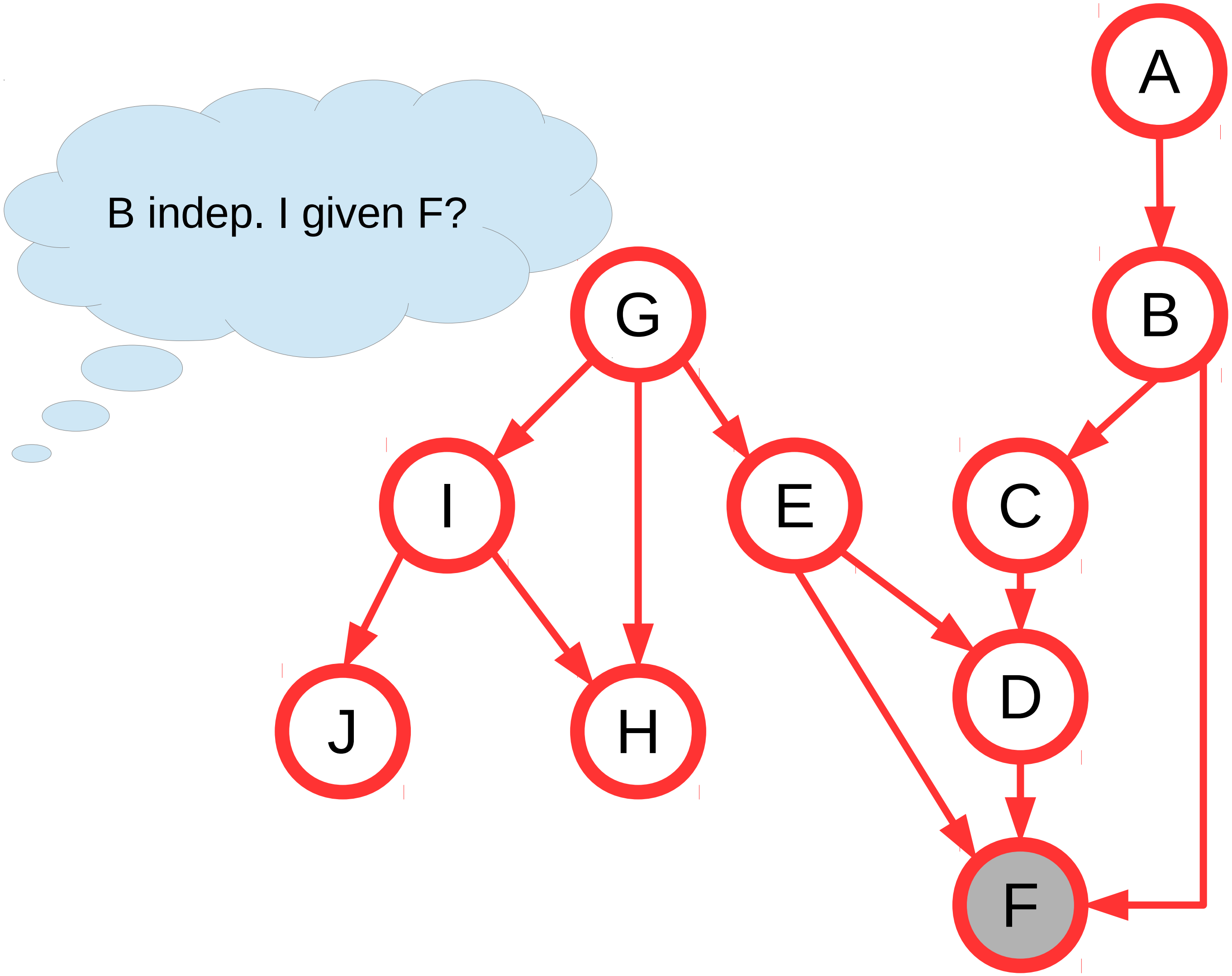
Solutions homework #2

Carlos Cotrini
October 27, 2017

Probabilistic foundations of artificial intelligence

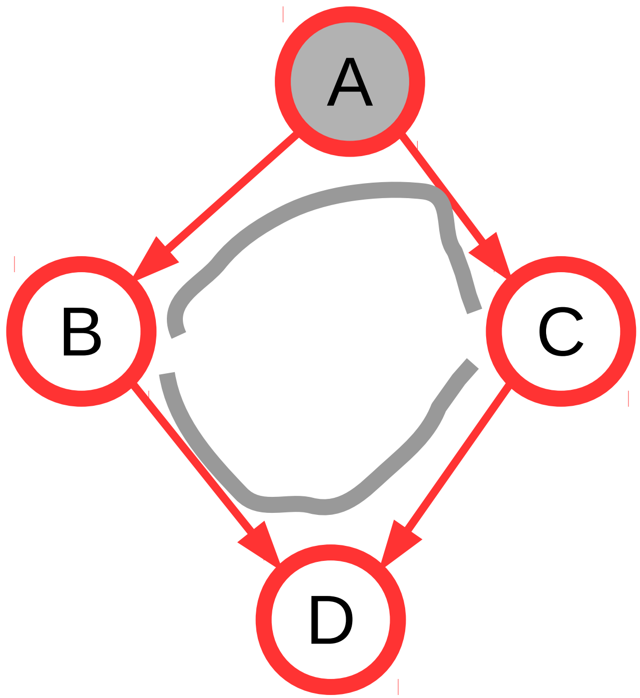
1. Bayesian Networks: d-separation

- Solutions task 1

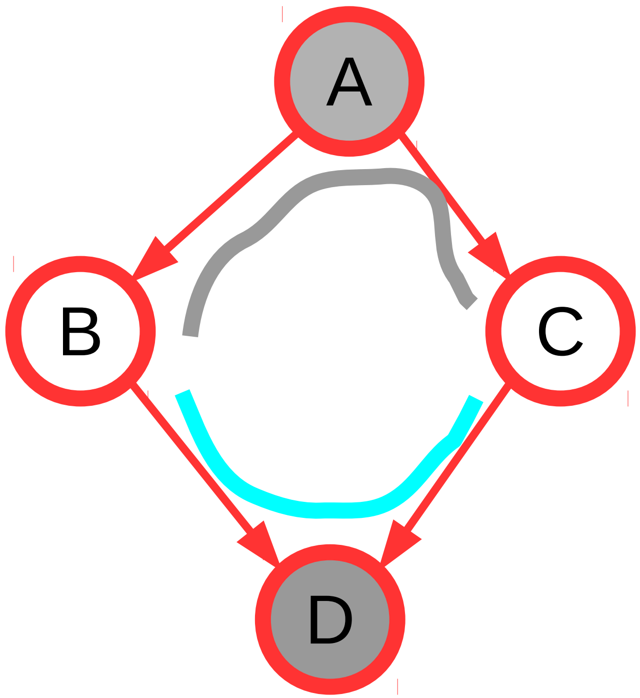


Principle of d-separation

- Given a set of observed variables, if there is no active trail between two variables, then they are independent.



B indep from C | A

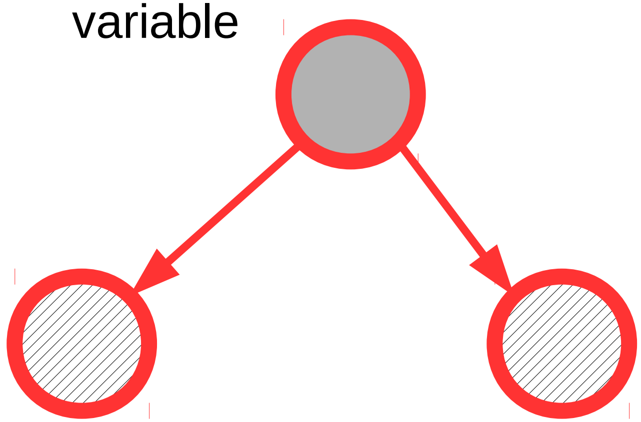


Unclear if B indep from C | A, D

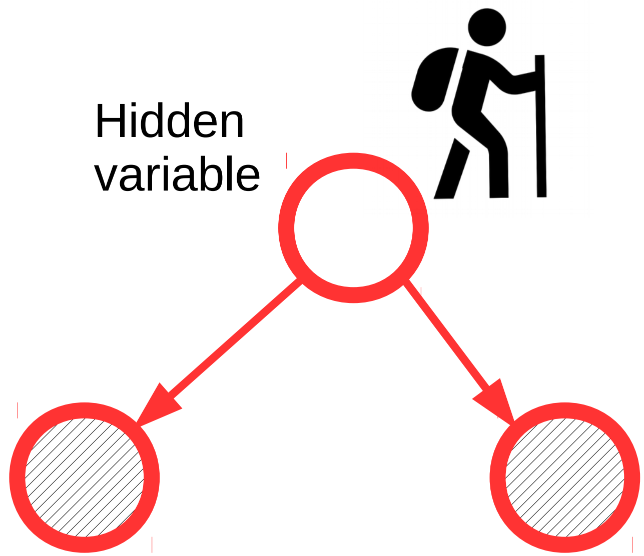
1. Bayesian Networks: d-separation

- Recap on active trails. Case 1. (Mountain)

Observed variable

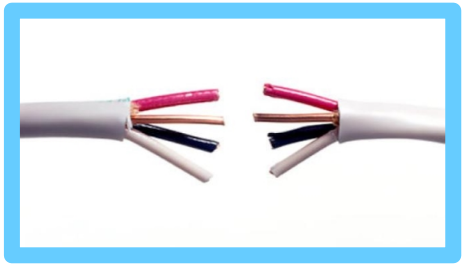


Hidden variable



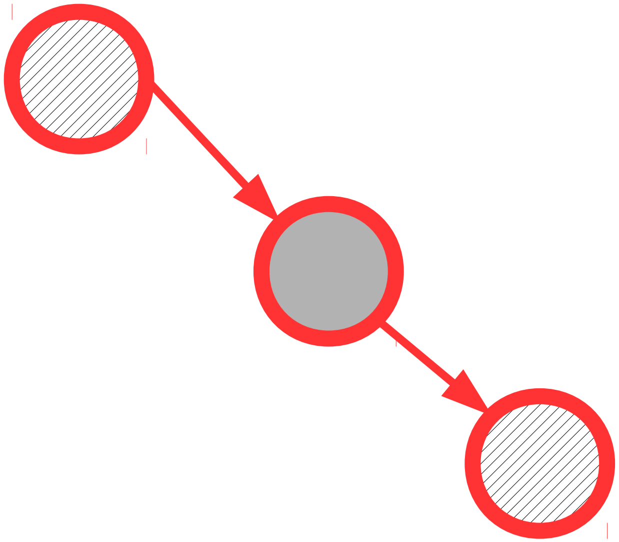
Observed or hidden variable

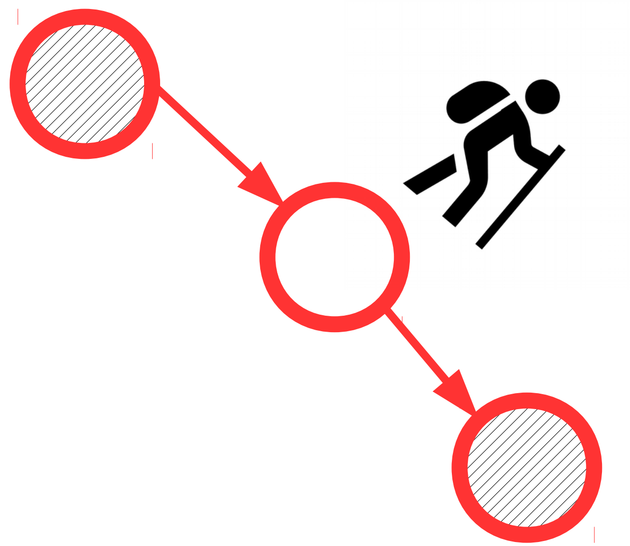
Inactive!



1. Bayesian Networks: d-separation

- Recap on active trails. Case 2a. (Downhill)





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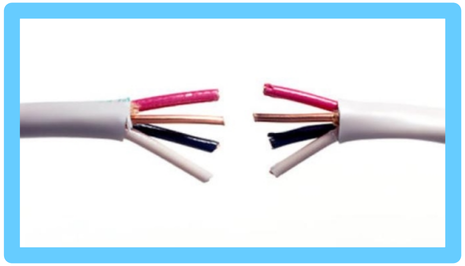
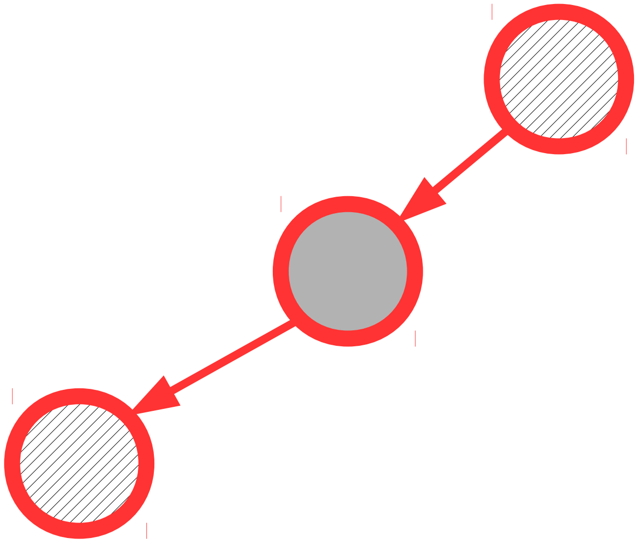
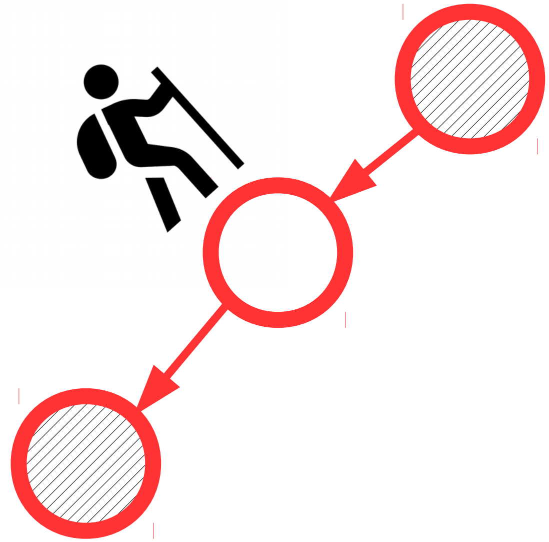


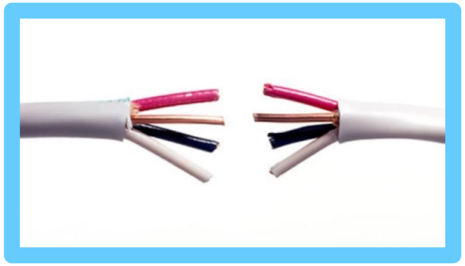
1. Bayesian Networks: d-separation

- Recap on active trails. Case 2b. (Uphill)



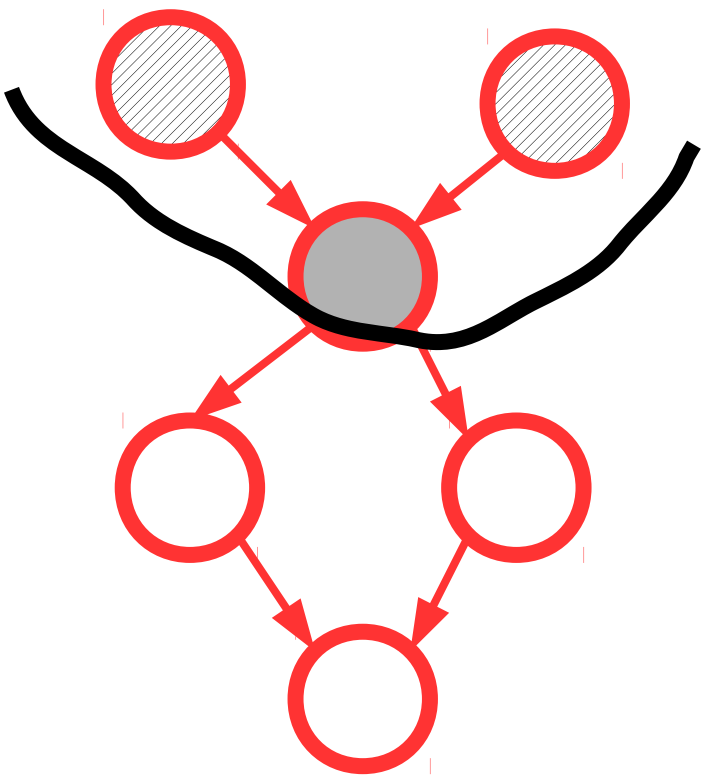


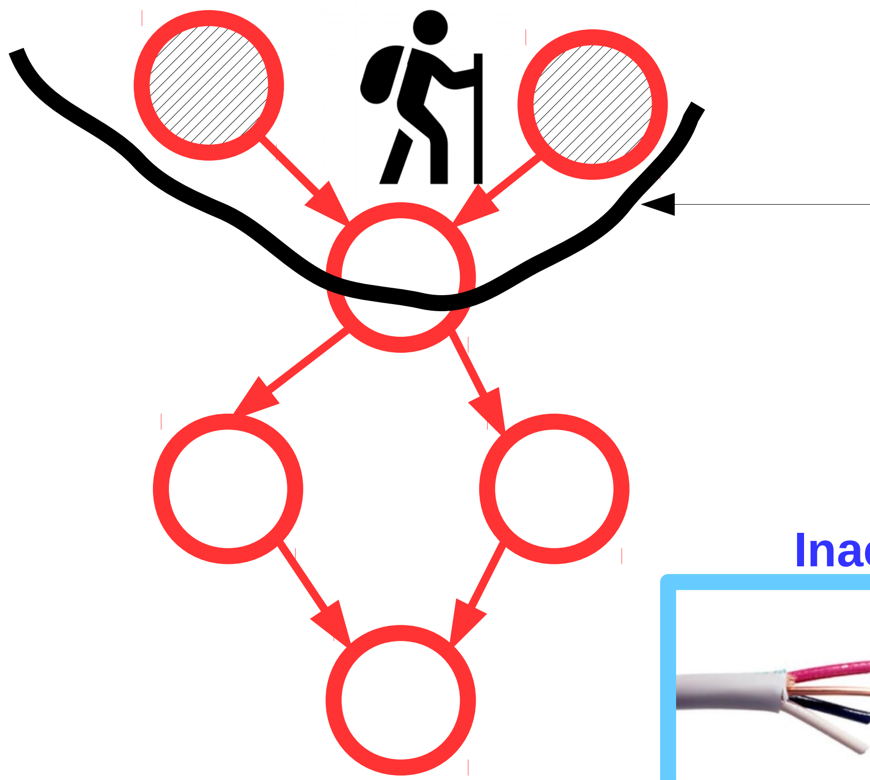
Inactive!



1. Bayesian Networks: d-separation

- Recap on active trails. Case 3. (Valley)

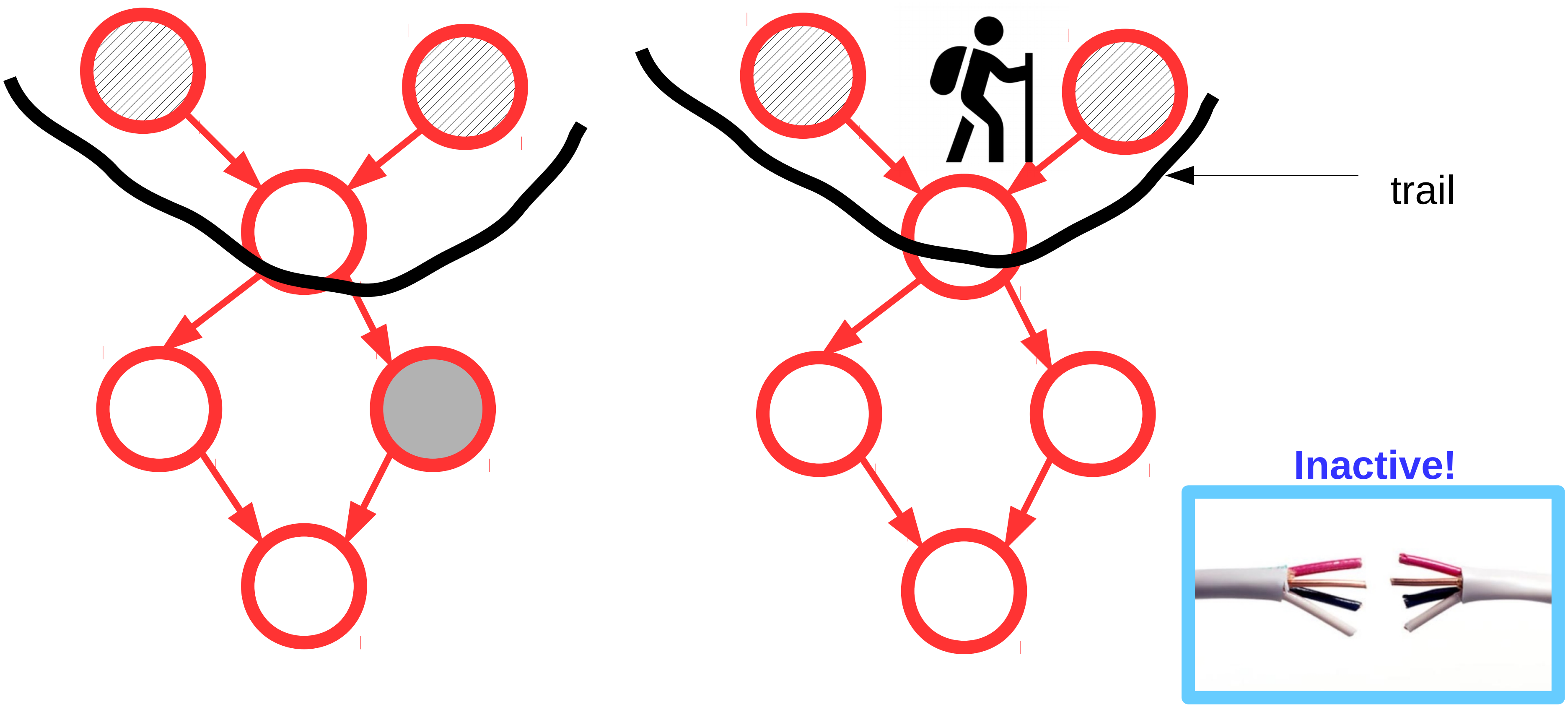




Inactive!

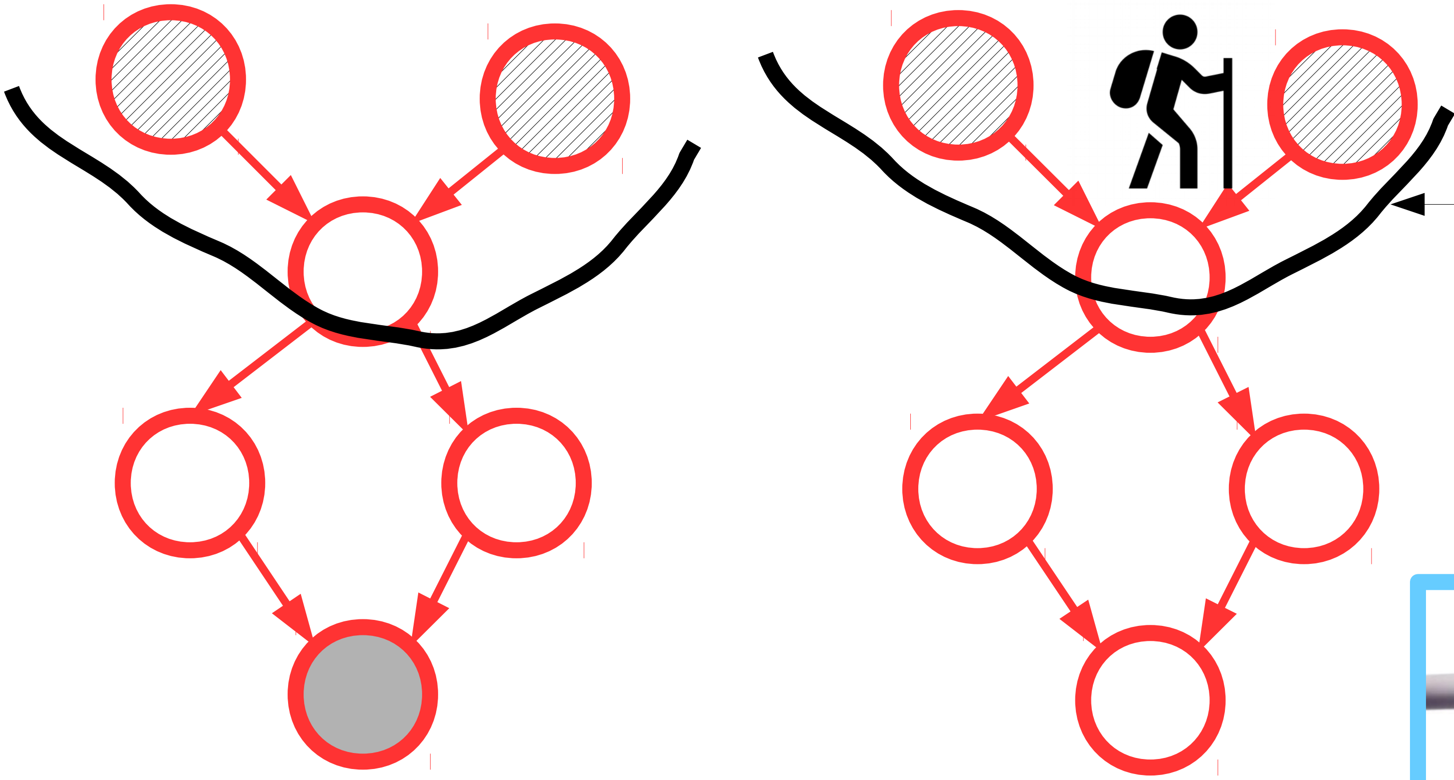
1. Bayesian Networks: d-separation

- Recap on active trails. Case 3.



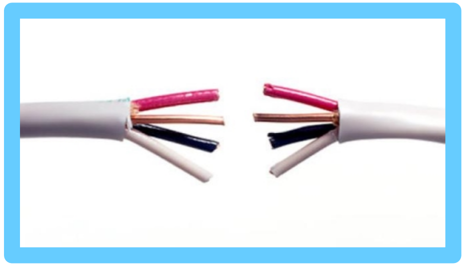
1. Bayesian Networks: d-separation

- Recap on active trails. Case 3.



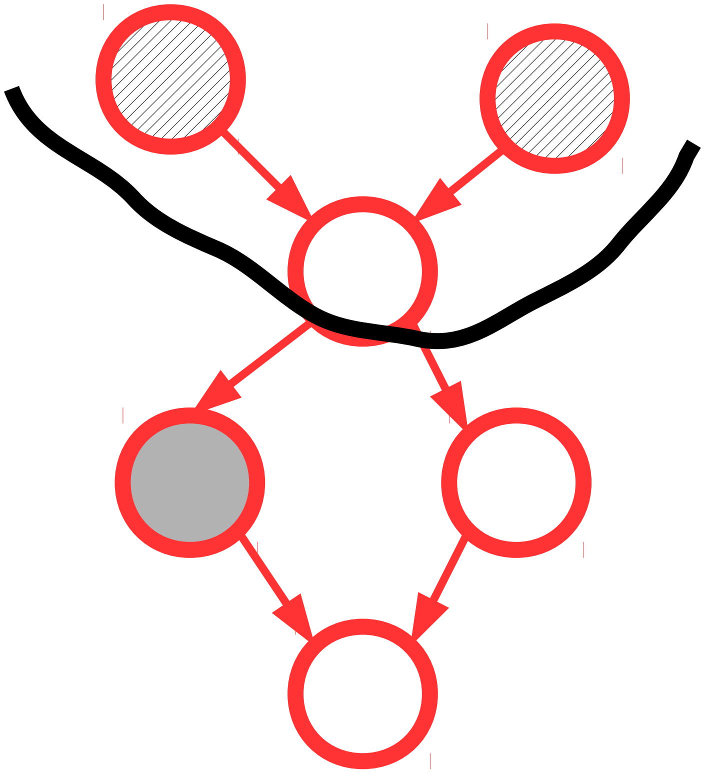
trail

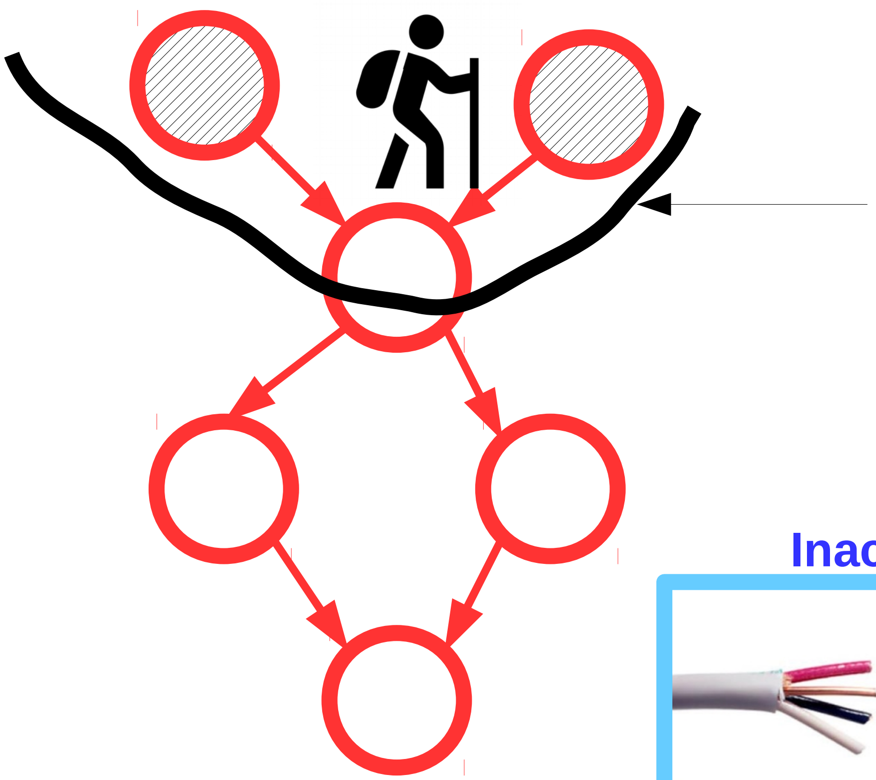
Inactive!



1. Bayesian Networks: d-separation

- Recap on active trails. Case 3.





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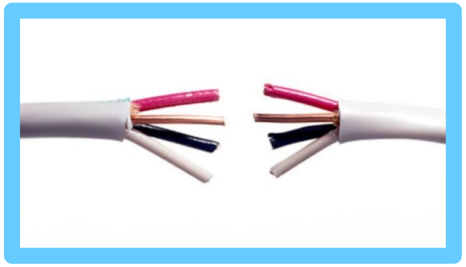
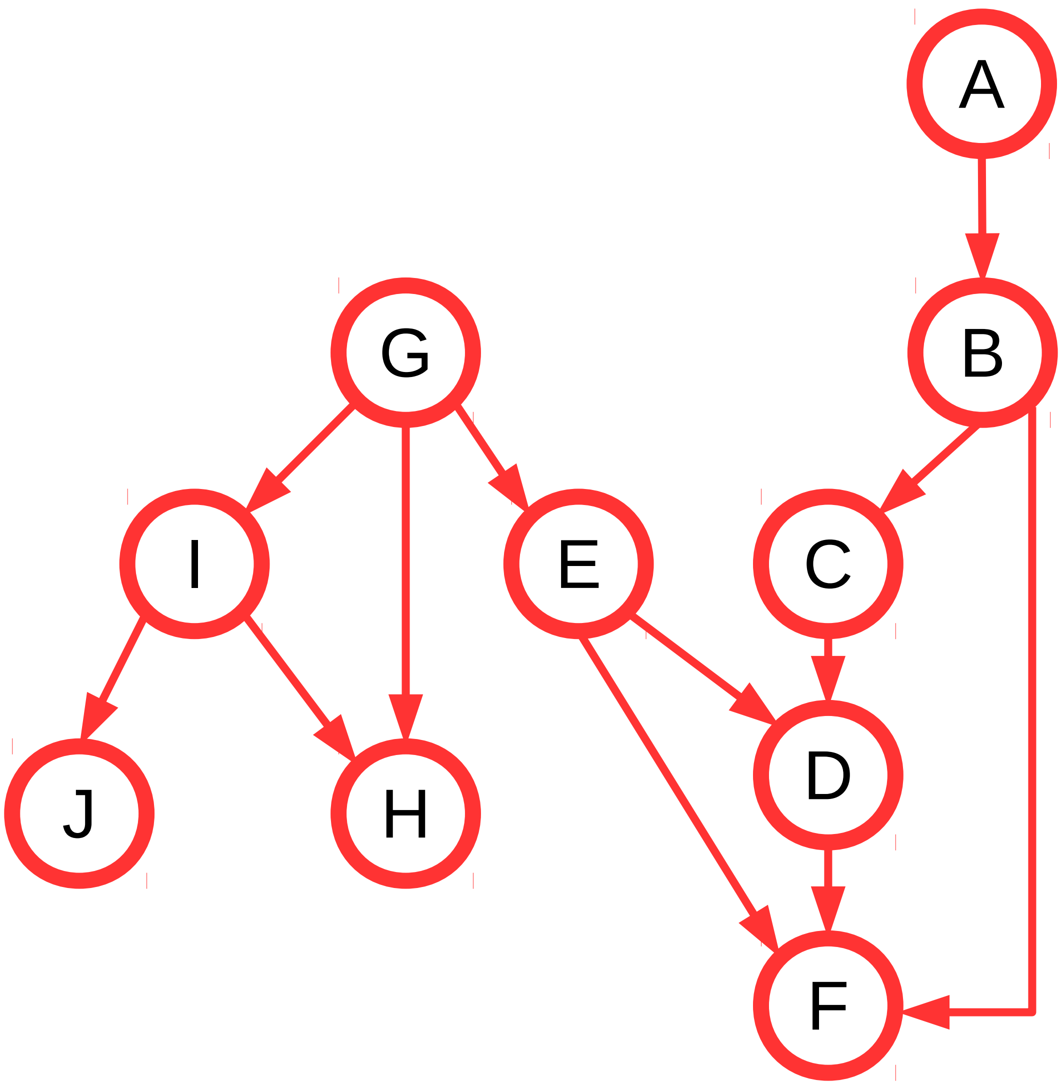


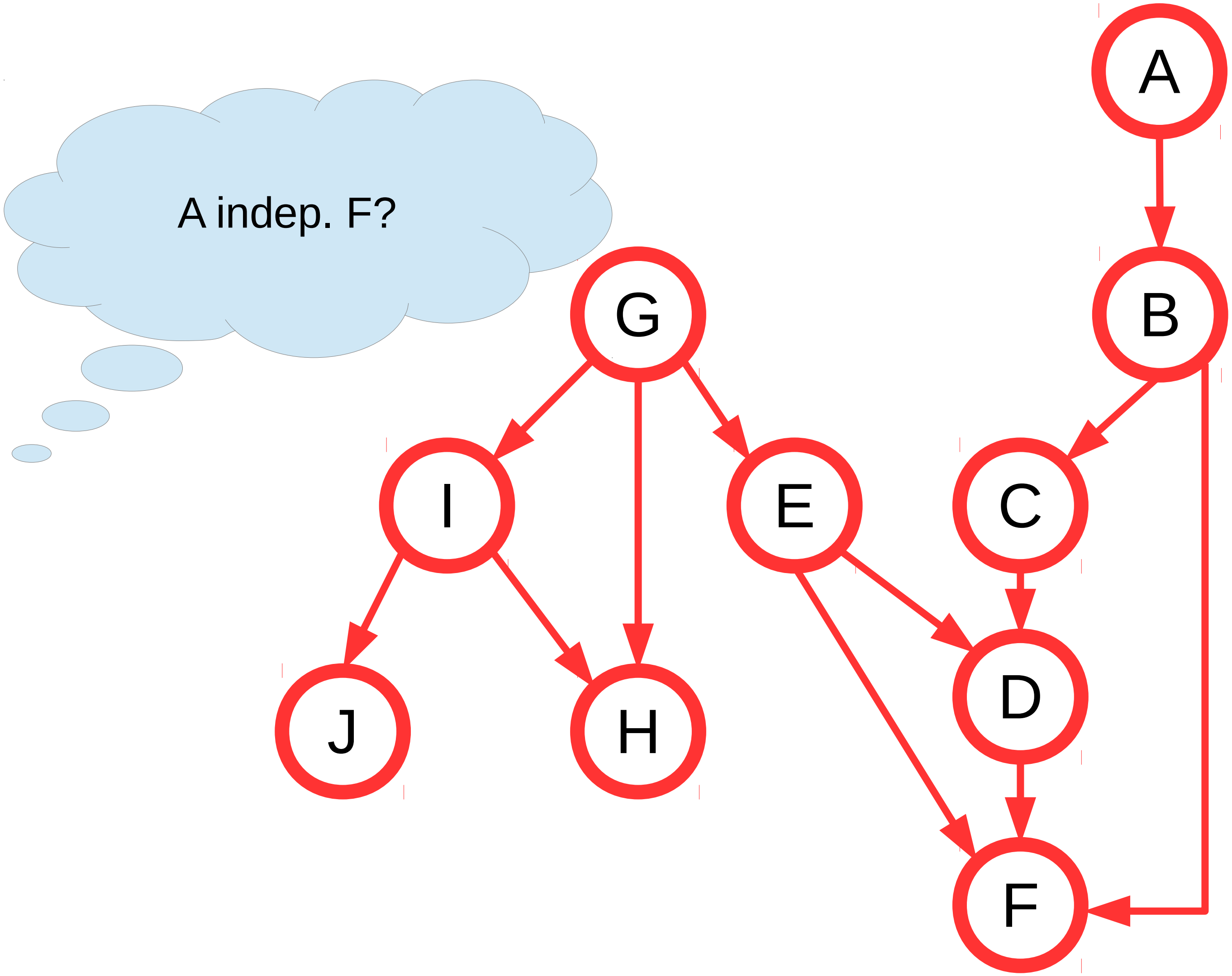
1. Bayesian Networks: d-separation

- Solutions task 1



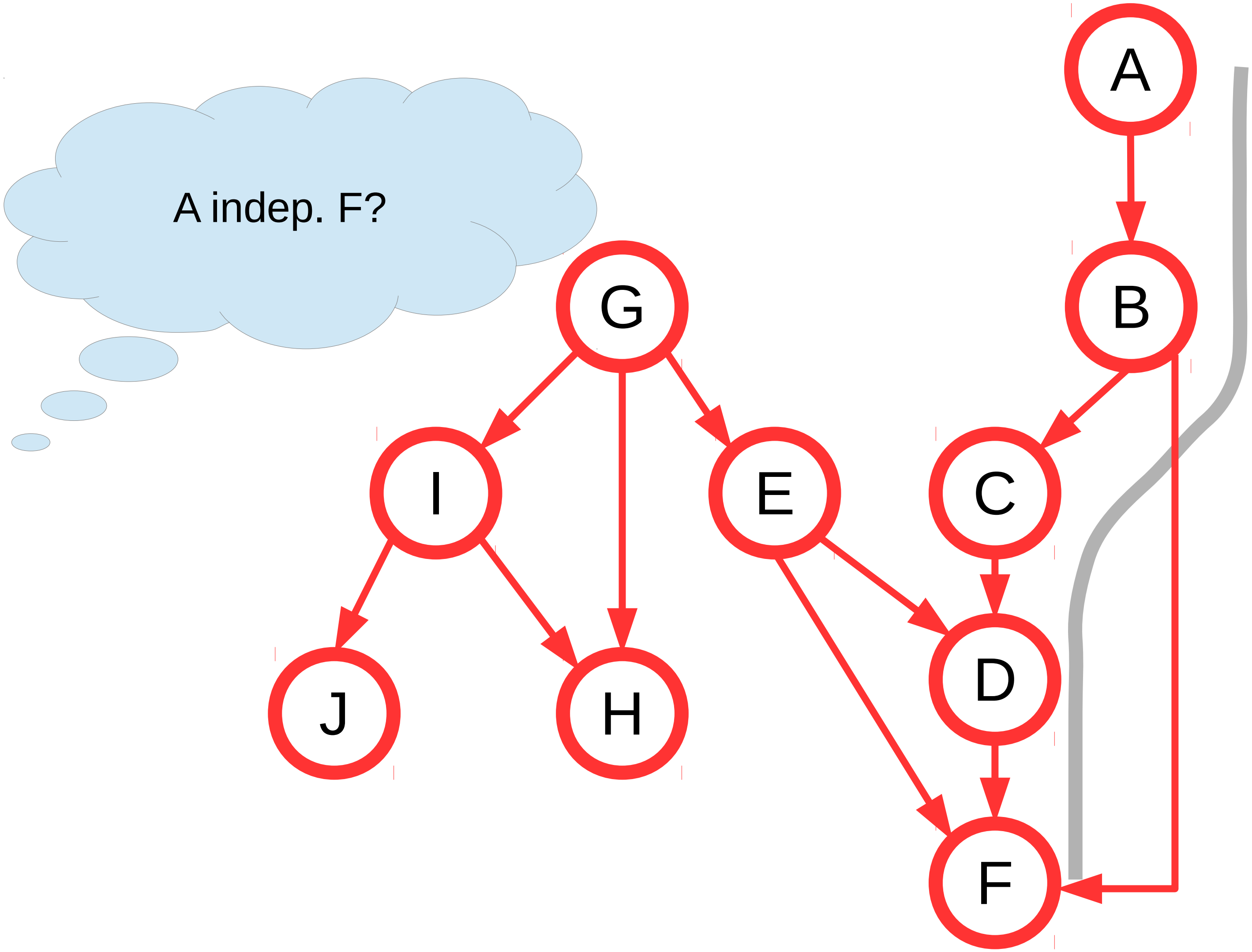
1. Bayesian Networks: d-separation

- Solutions task 1



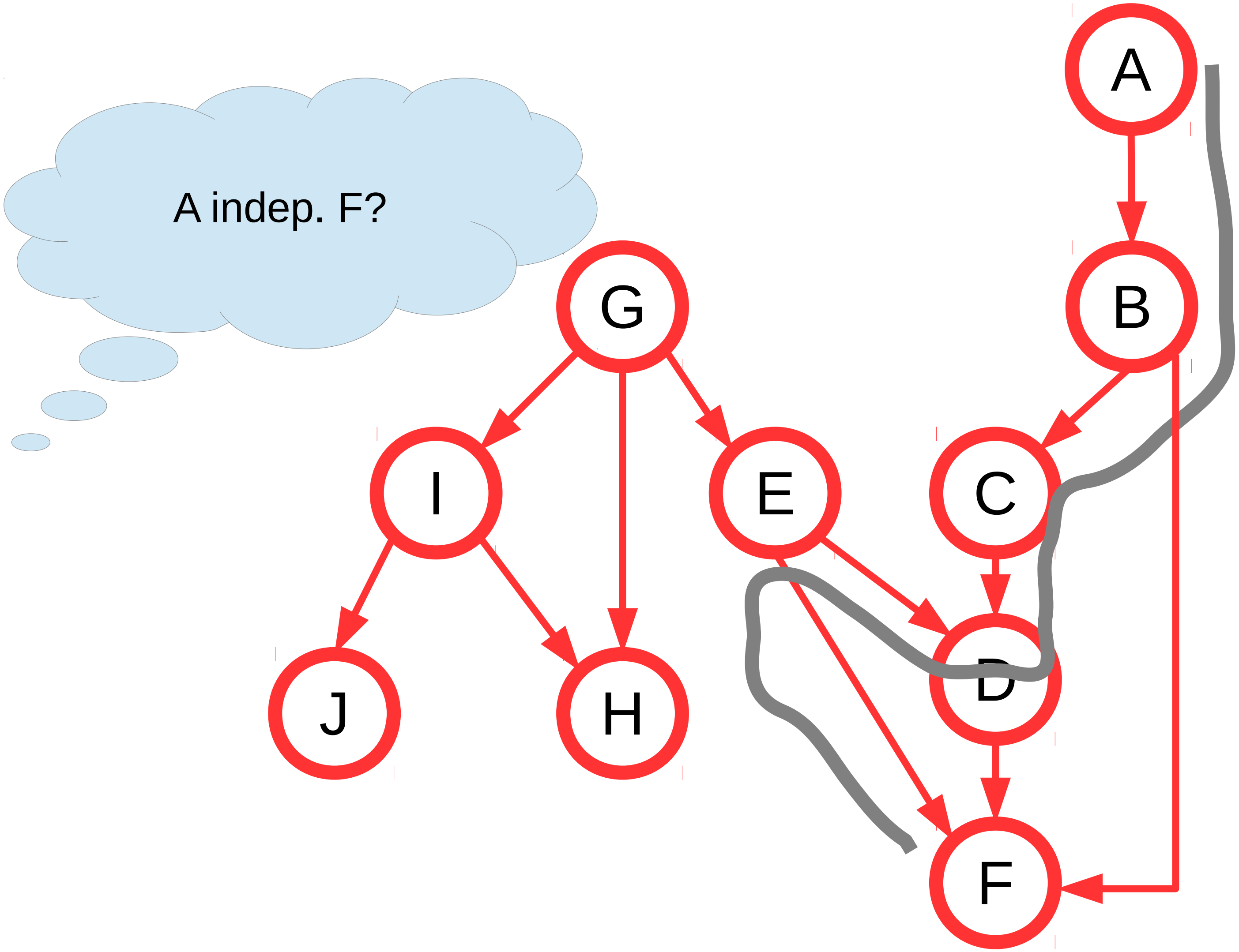
1. Bayesian Networks: d-separation

- Solutions task 1



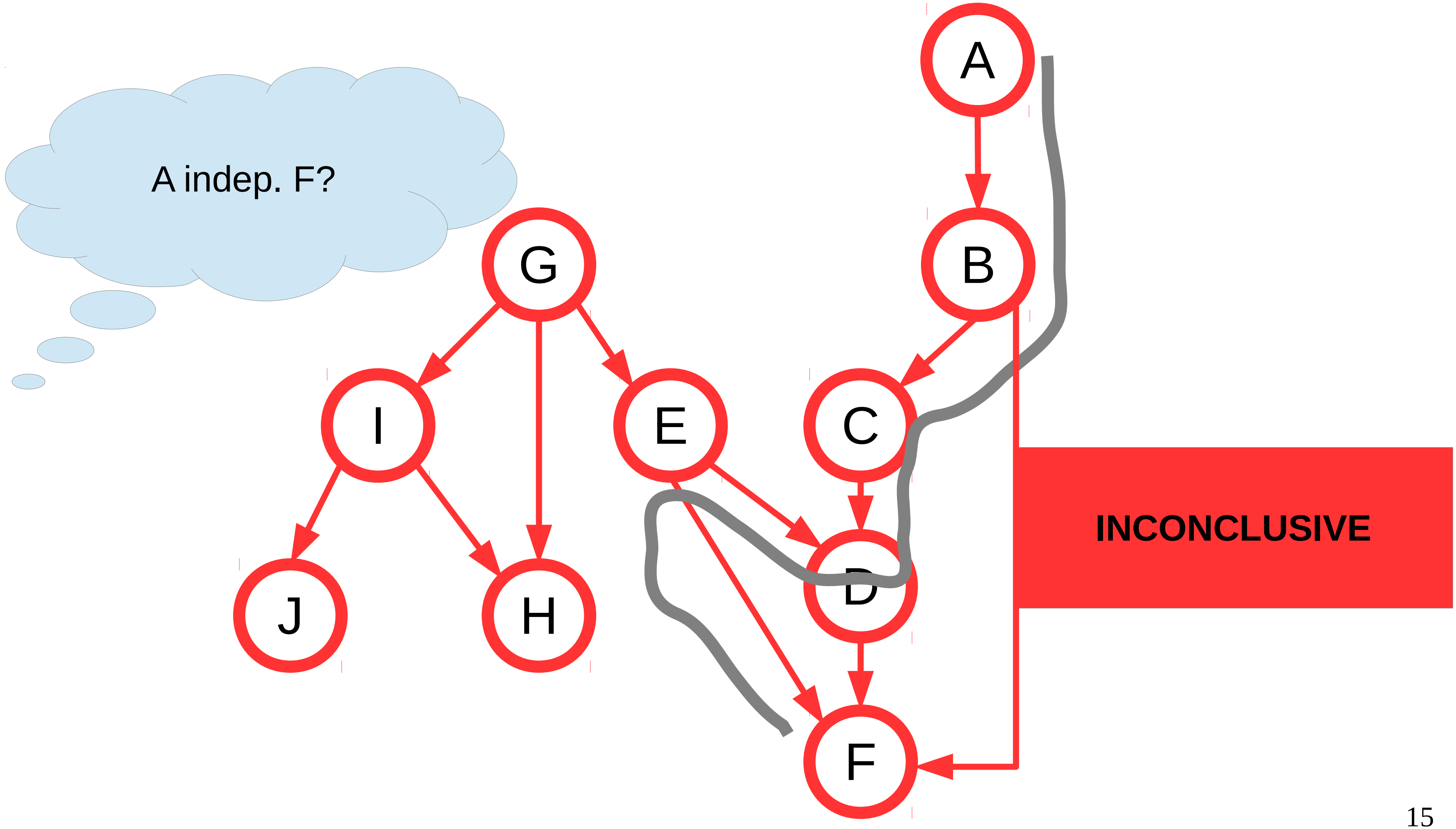
1. Bayesian Networks: d-separation

- Solutions task 1



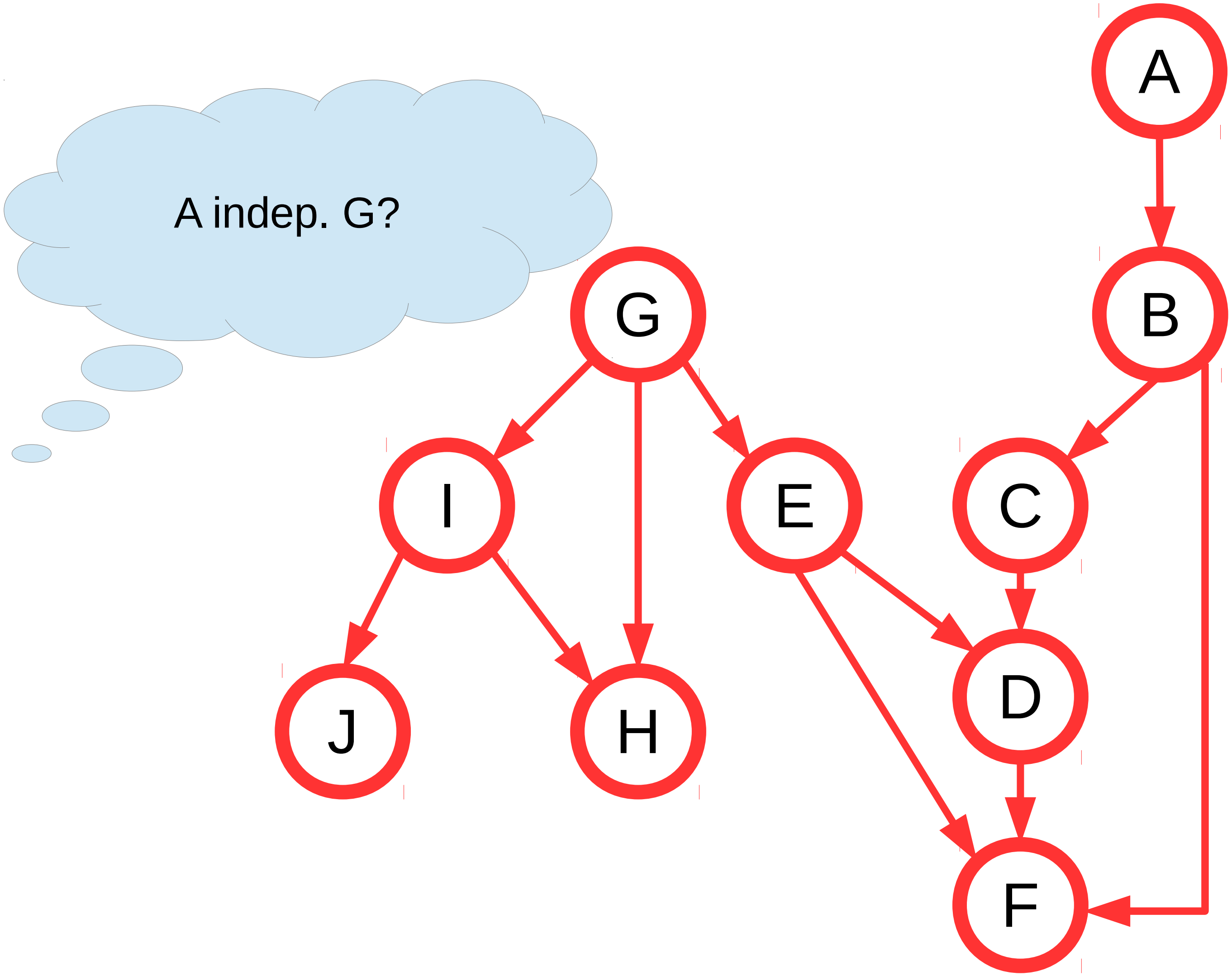
1. Bayesian Networks: d-separation

- Solutions task 1



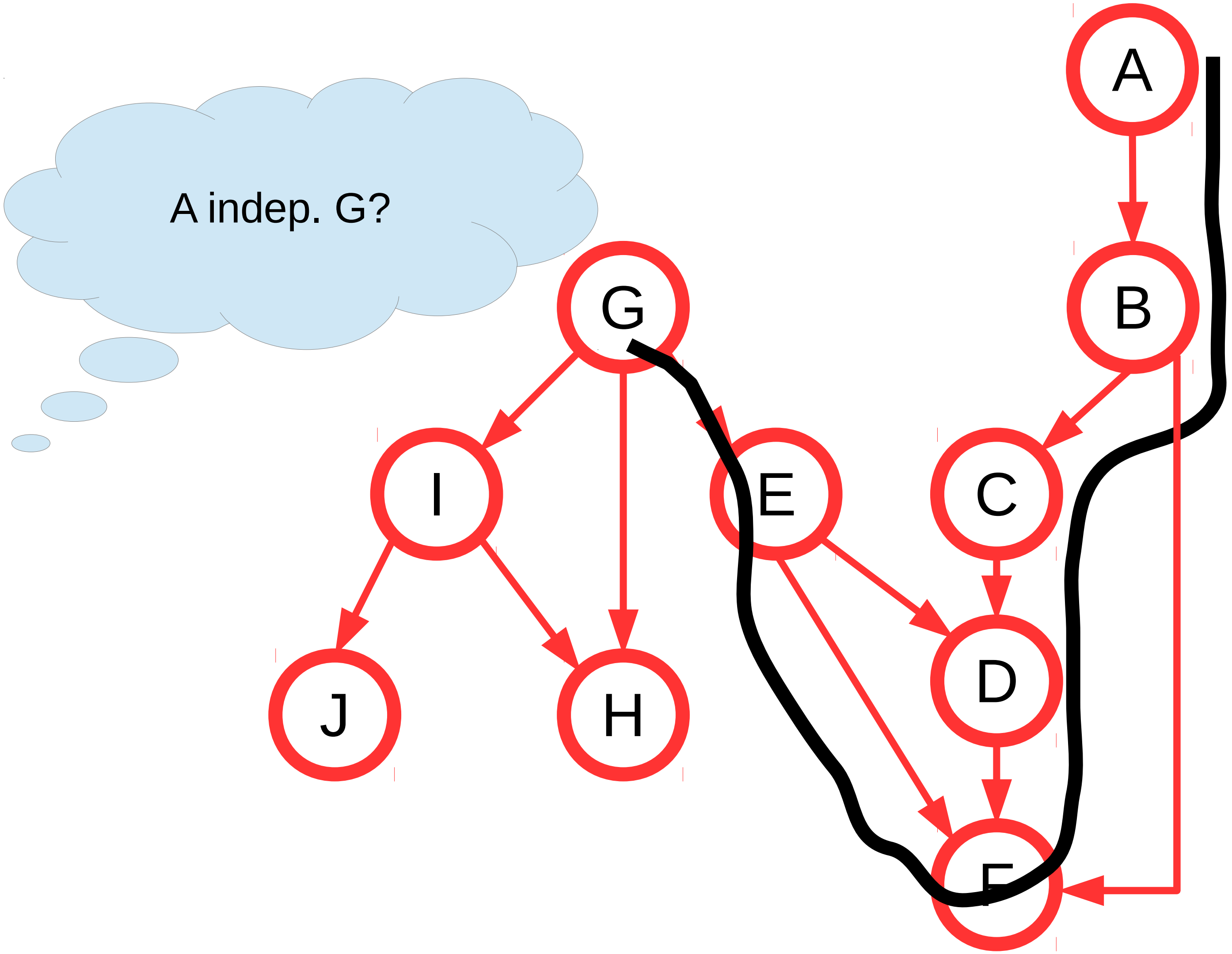
1. Bayesian Networks: d-separation

- Solutions task 1



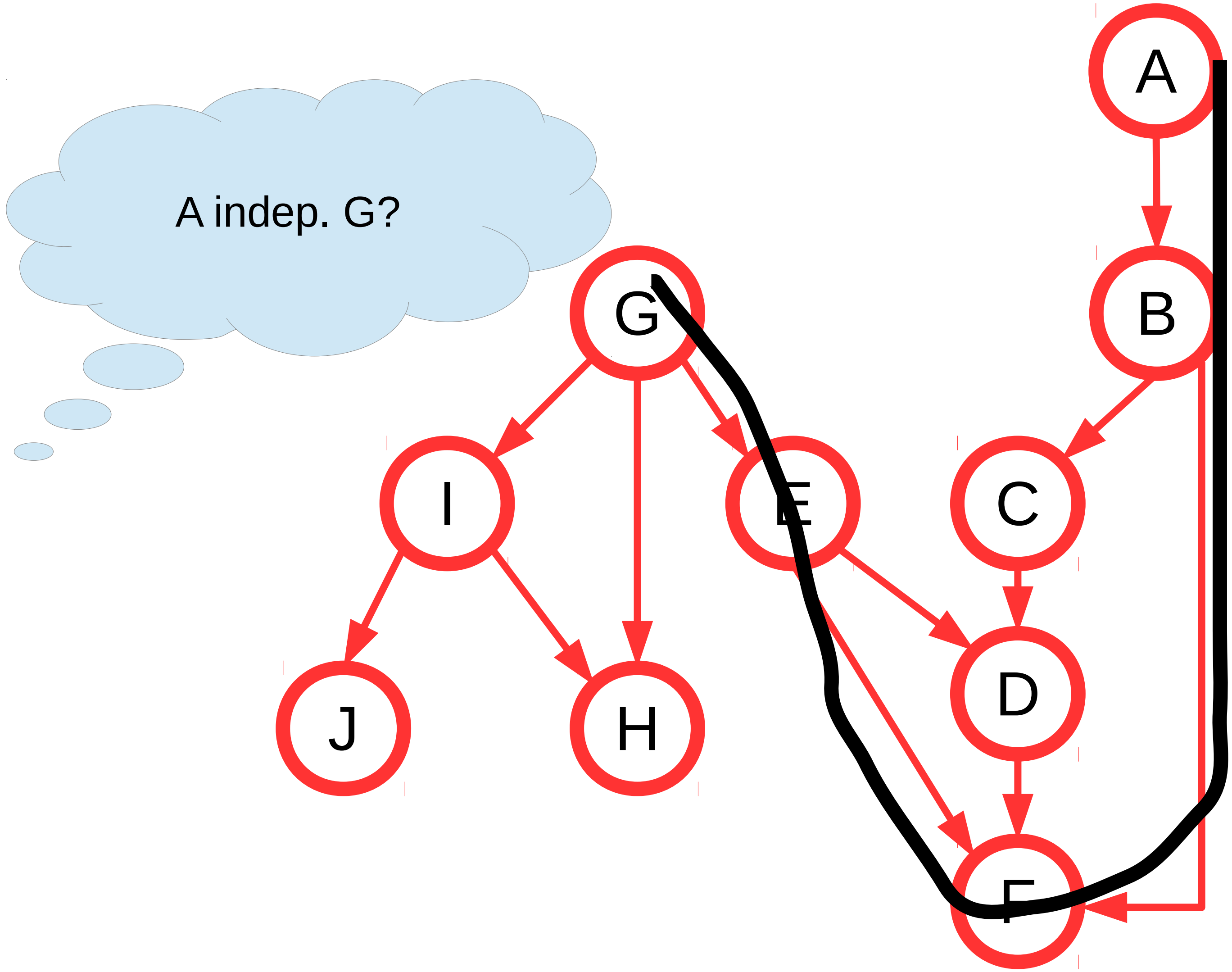
1. Bayesian Networks: d-separation

- Solutions task 1



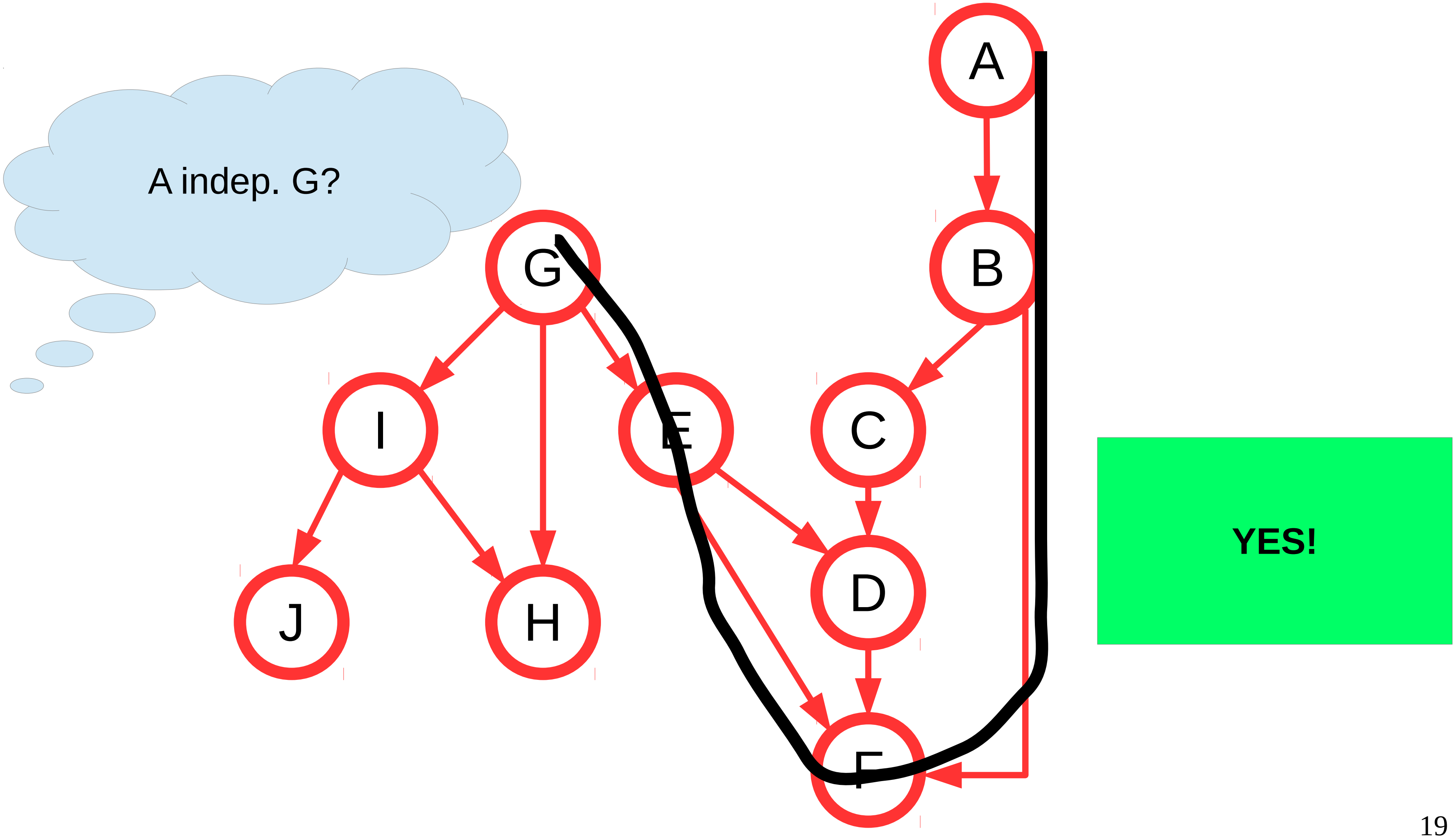
1. Bayesian Networks: d-separation

- Solutions task 1



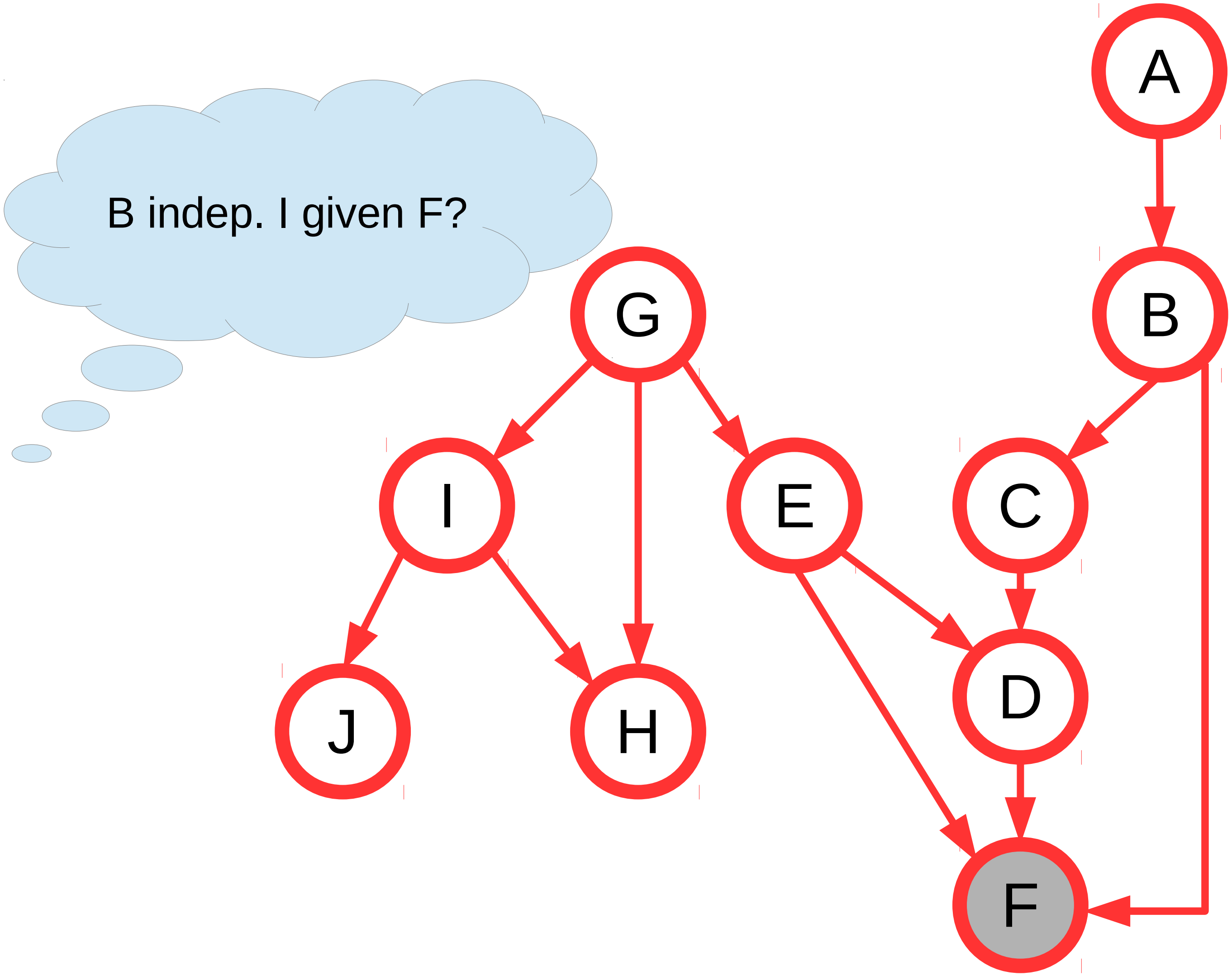
1. Bayesian Networks: d-separation

- Solutions task 1



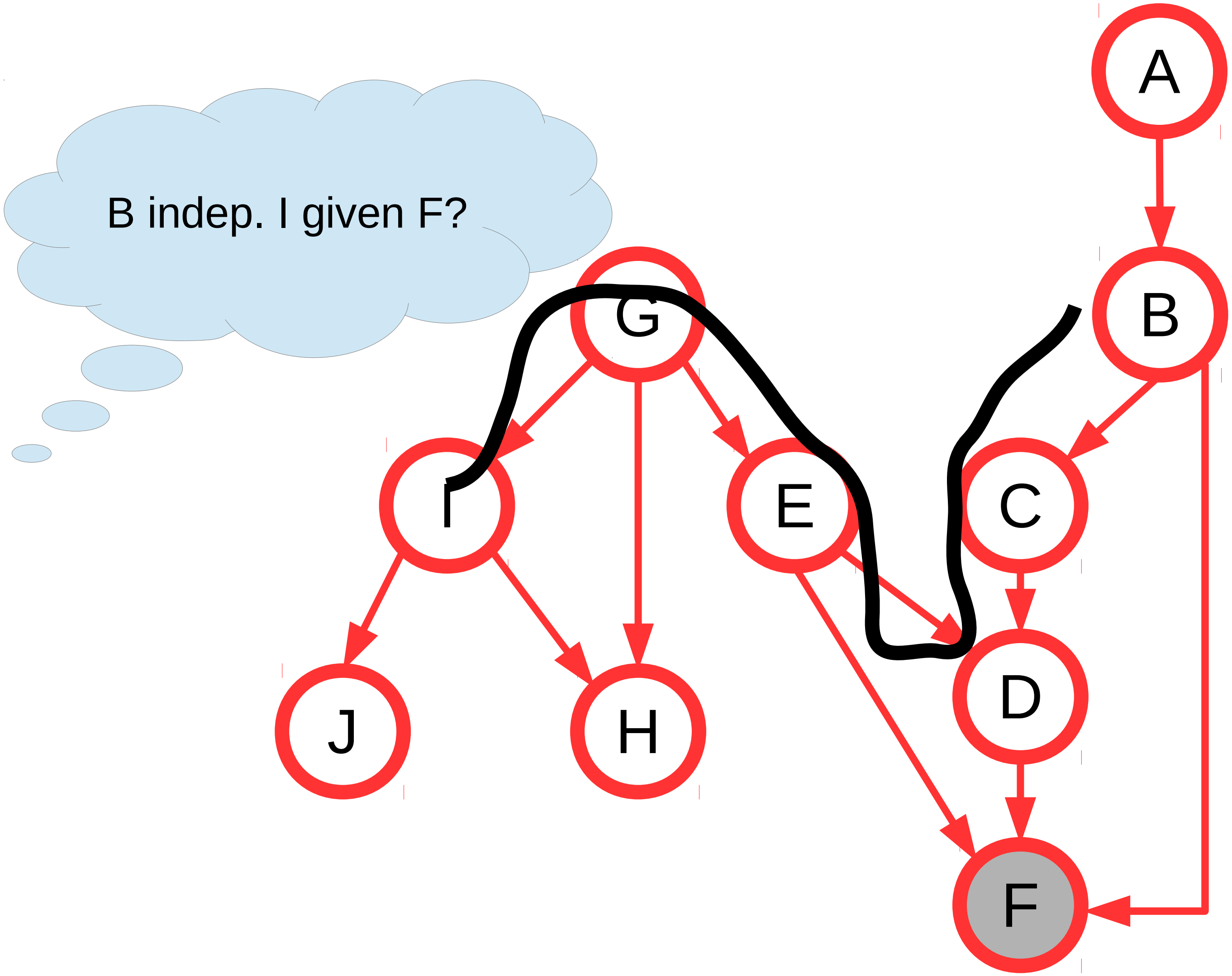
1. Bayesian Networks: d-separation

- Solutions task 1



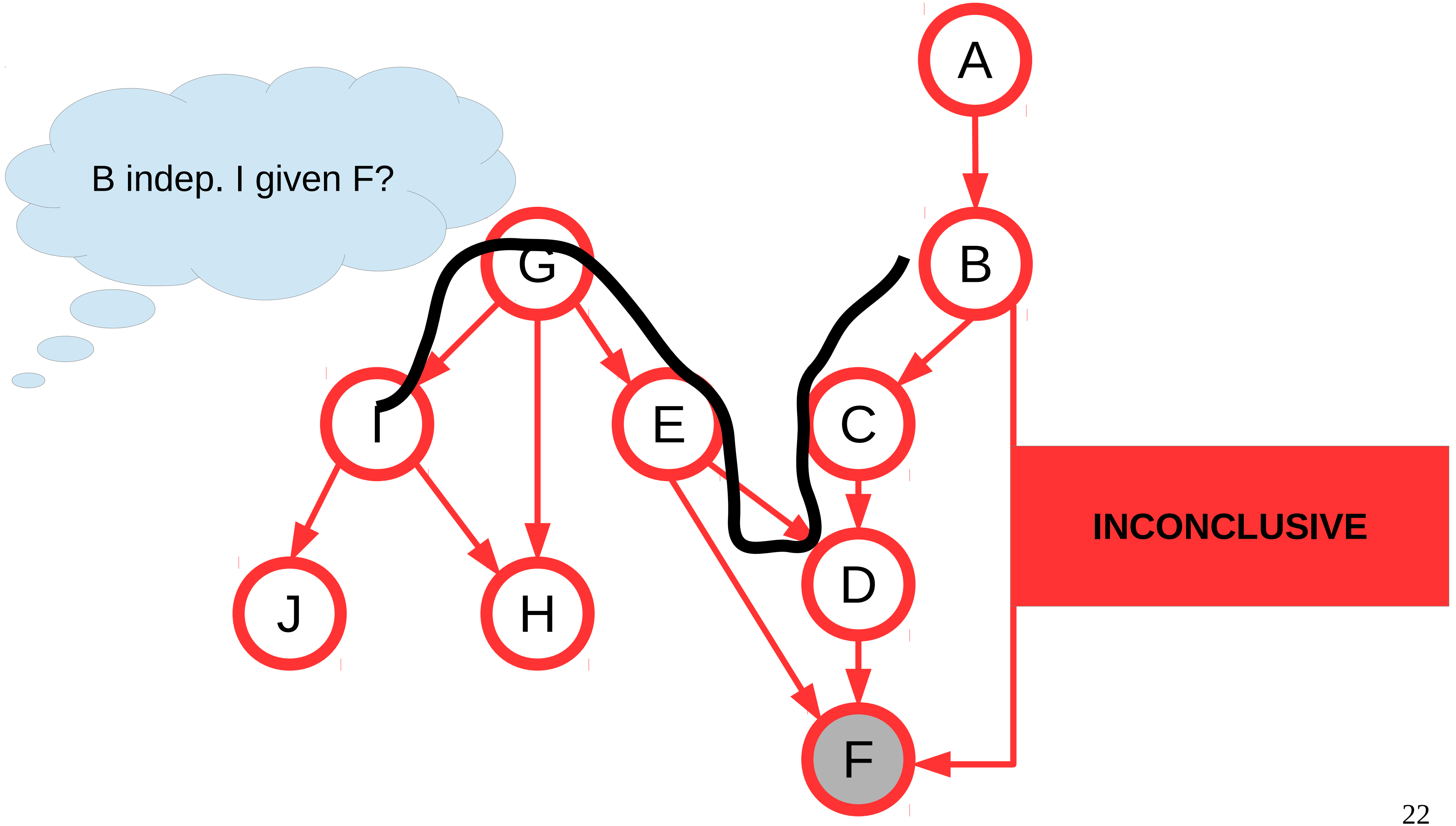
1. Bayesian Networks: d-separation

- Solutions task 1



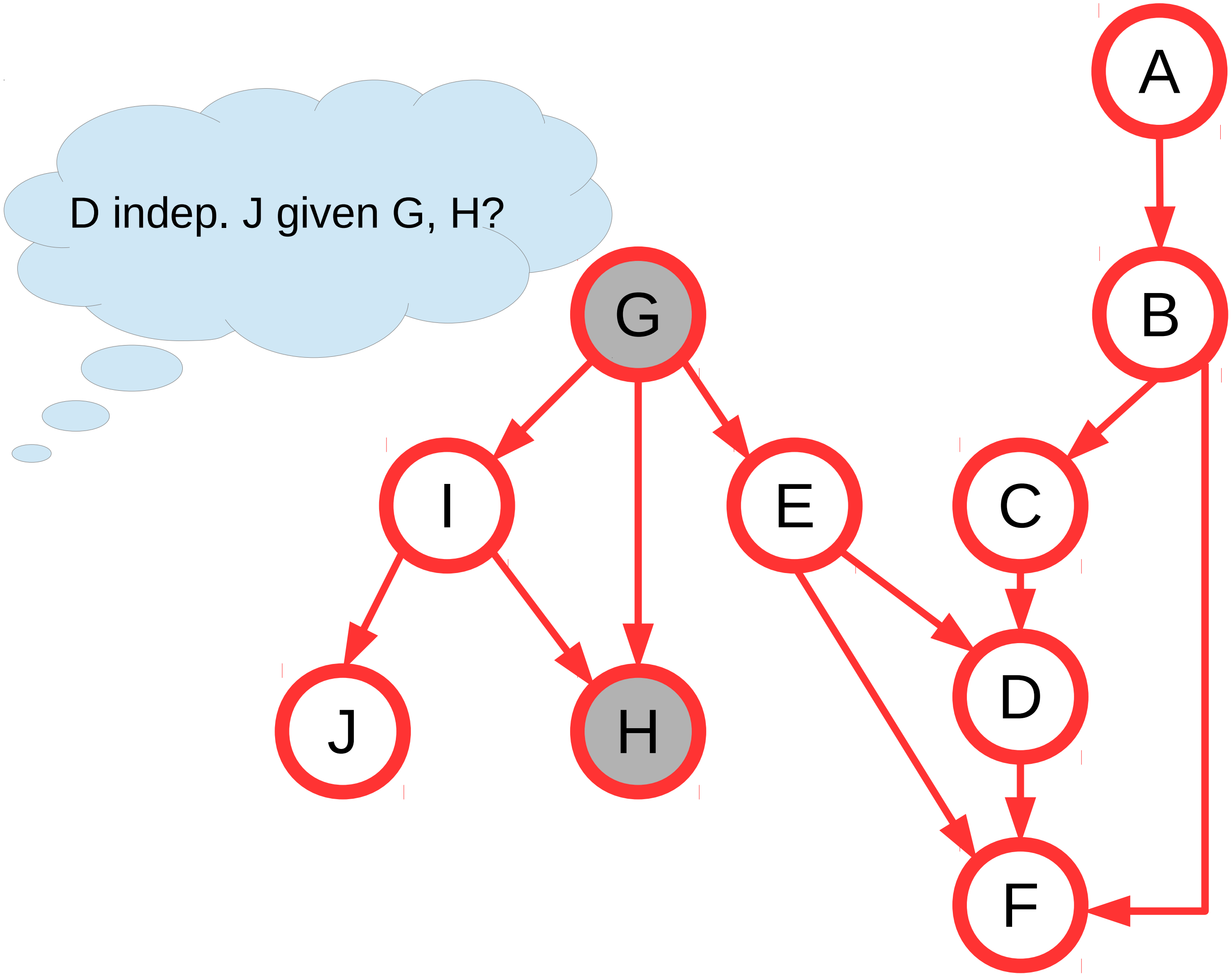
1. Bayesian Networks: d-separation

- Solutions task 1



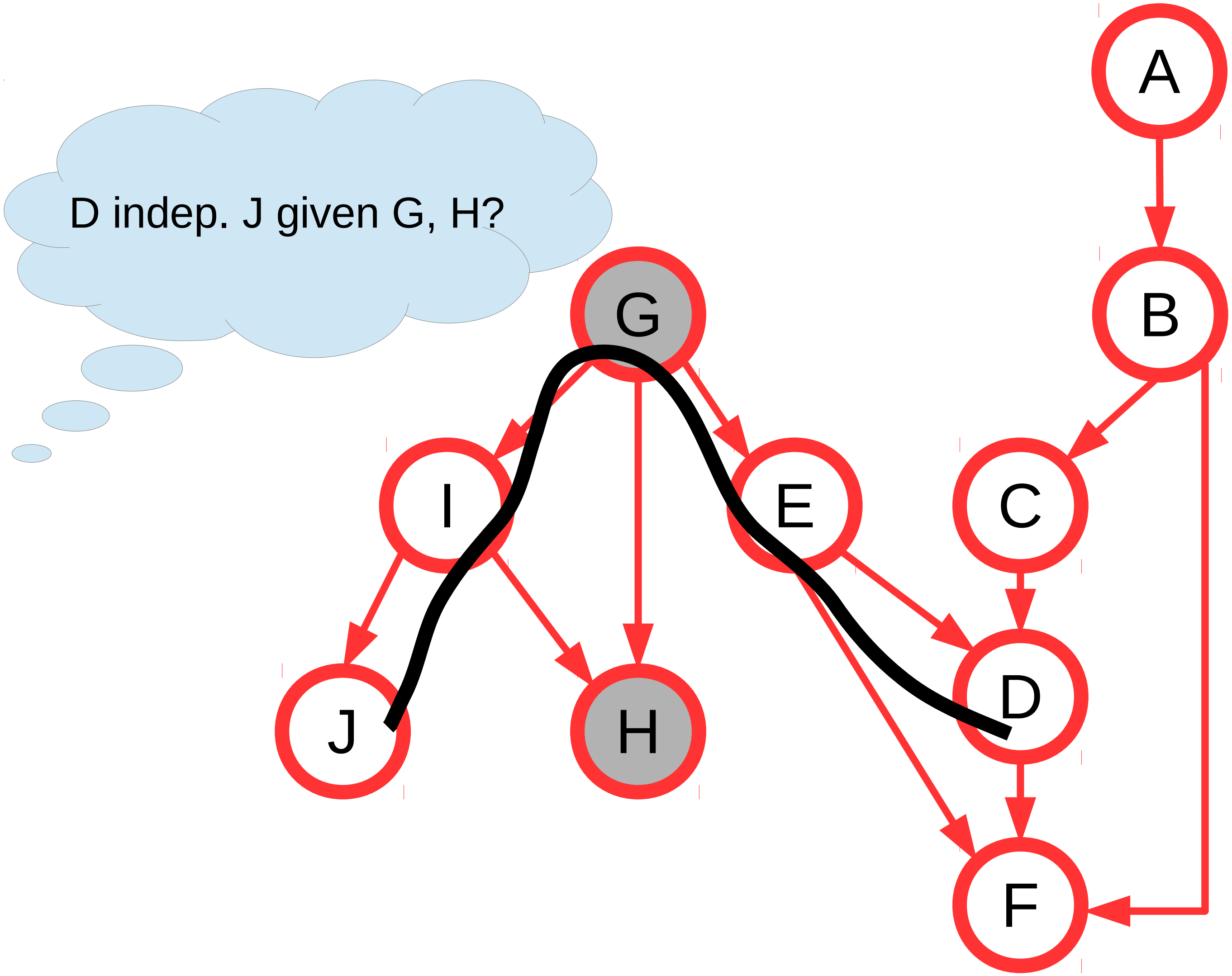
1. Bayesian Networks: d-separation

- Solutions task 1



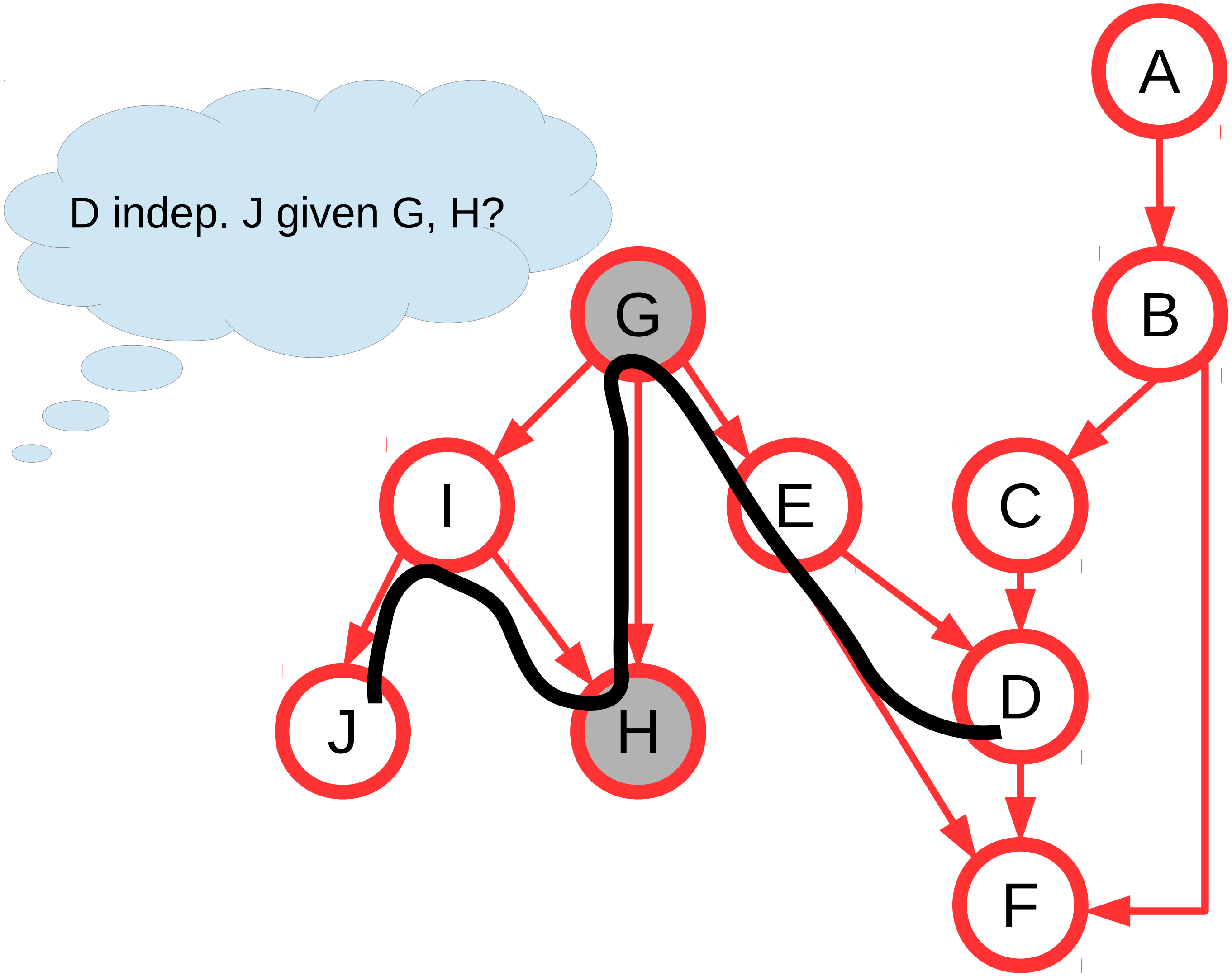
1. Bayesian Networks: d-separation

- Solutions task 1



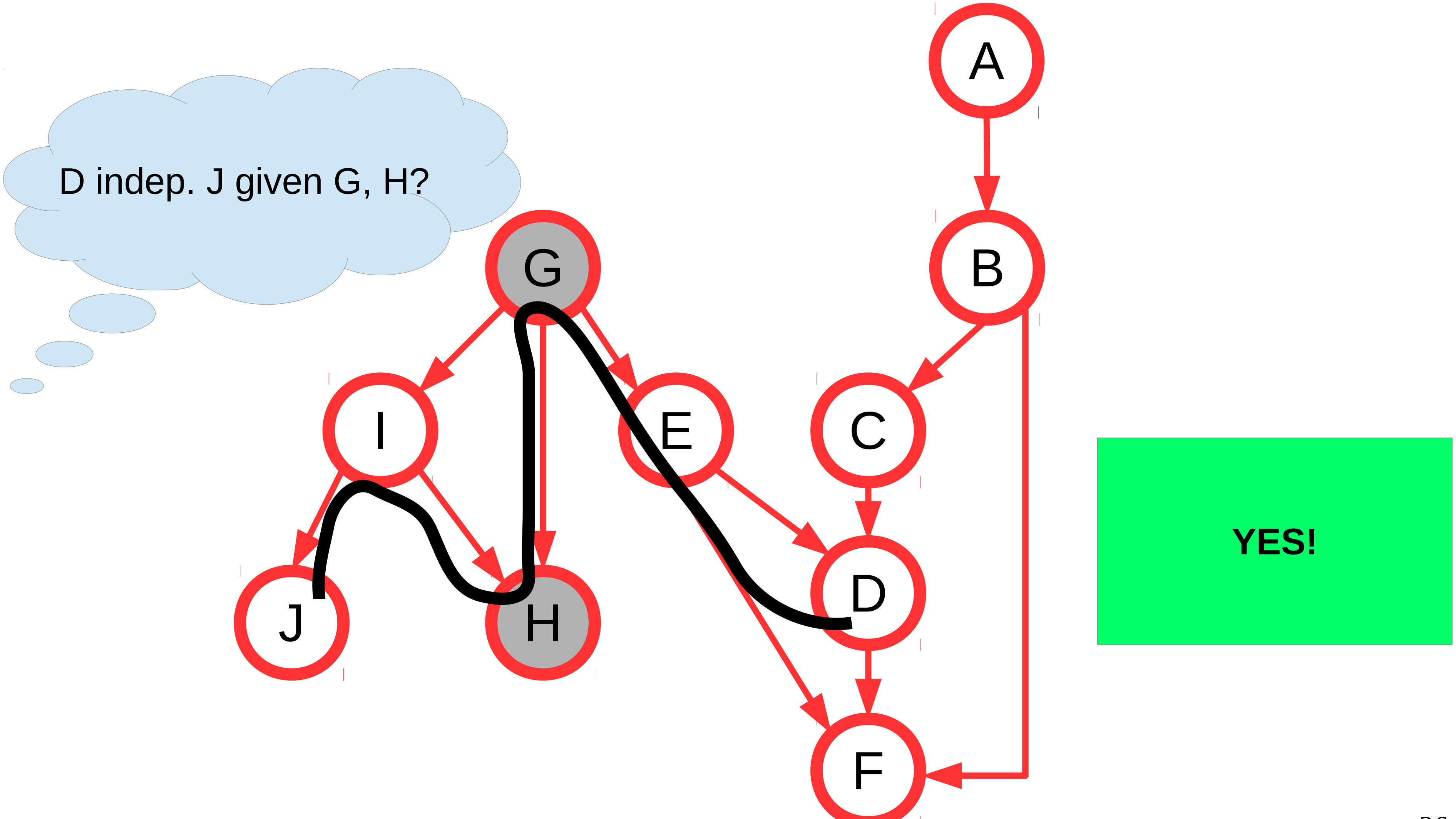
1. Bayesian Networks: d-separation

- Solutions task 1



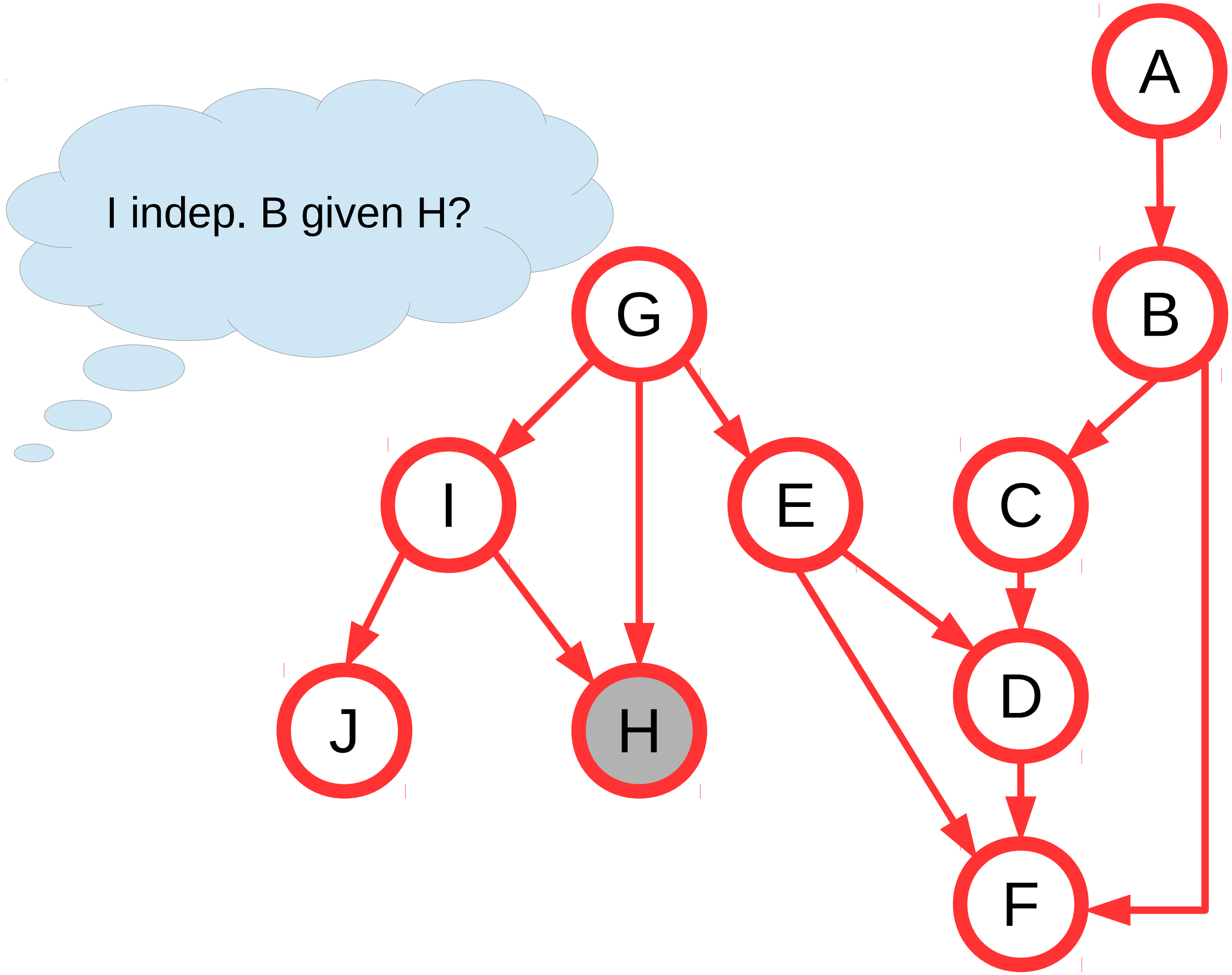
1. Bayesian Networks: d-separation

- Solutions task 1



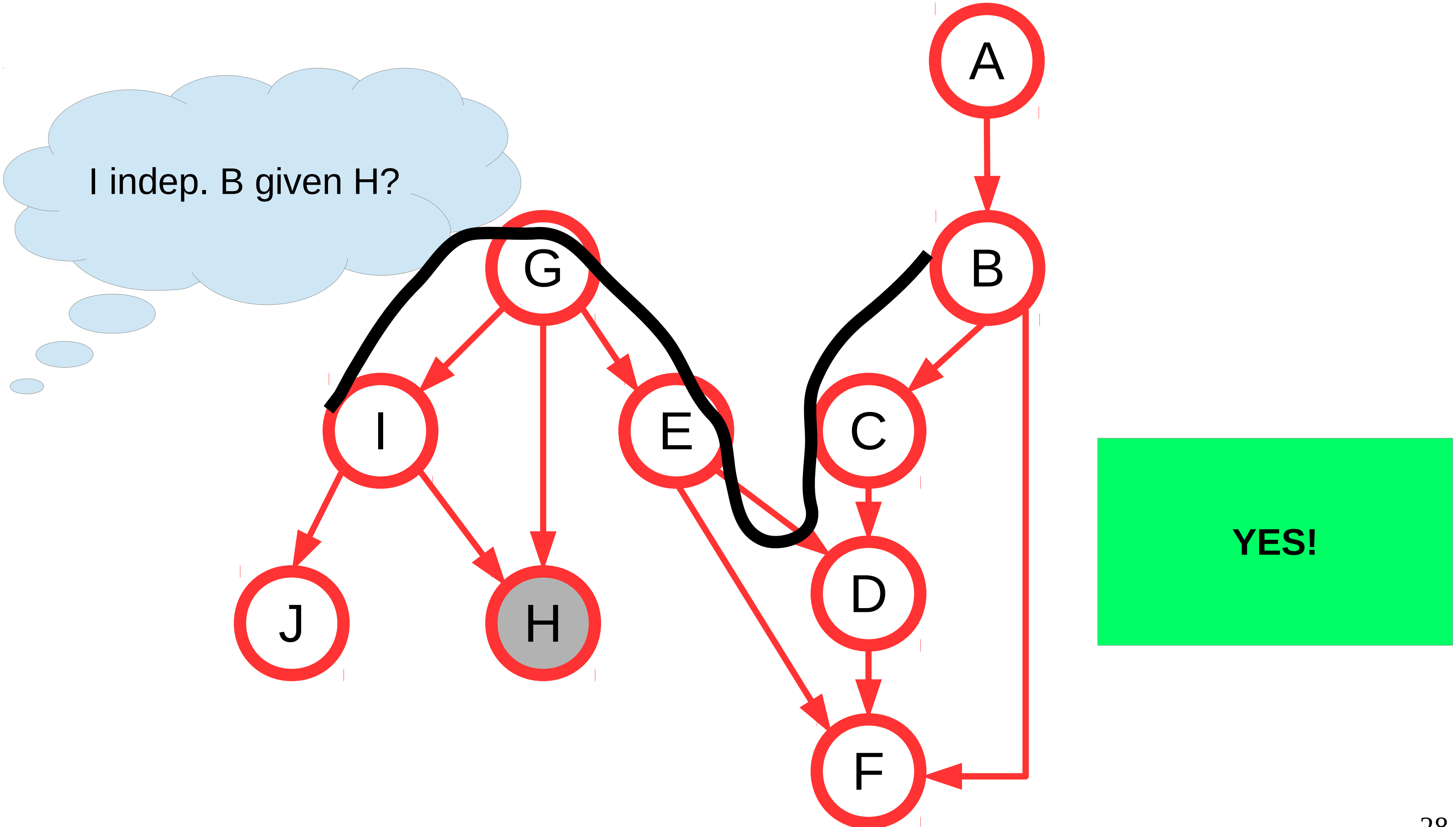
1. Bayesian Networks: d-separation

- Solutions task 1



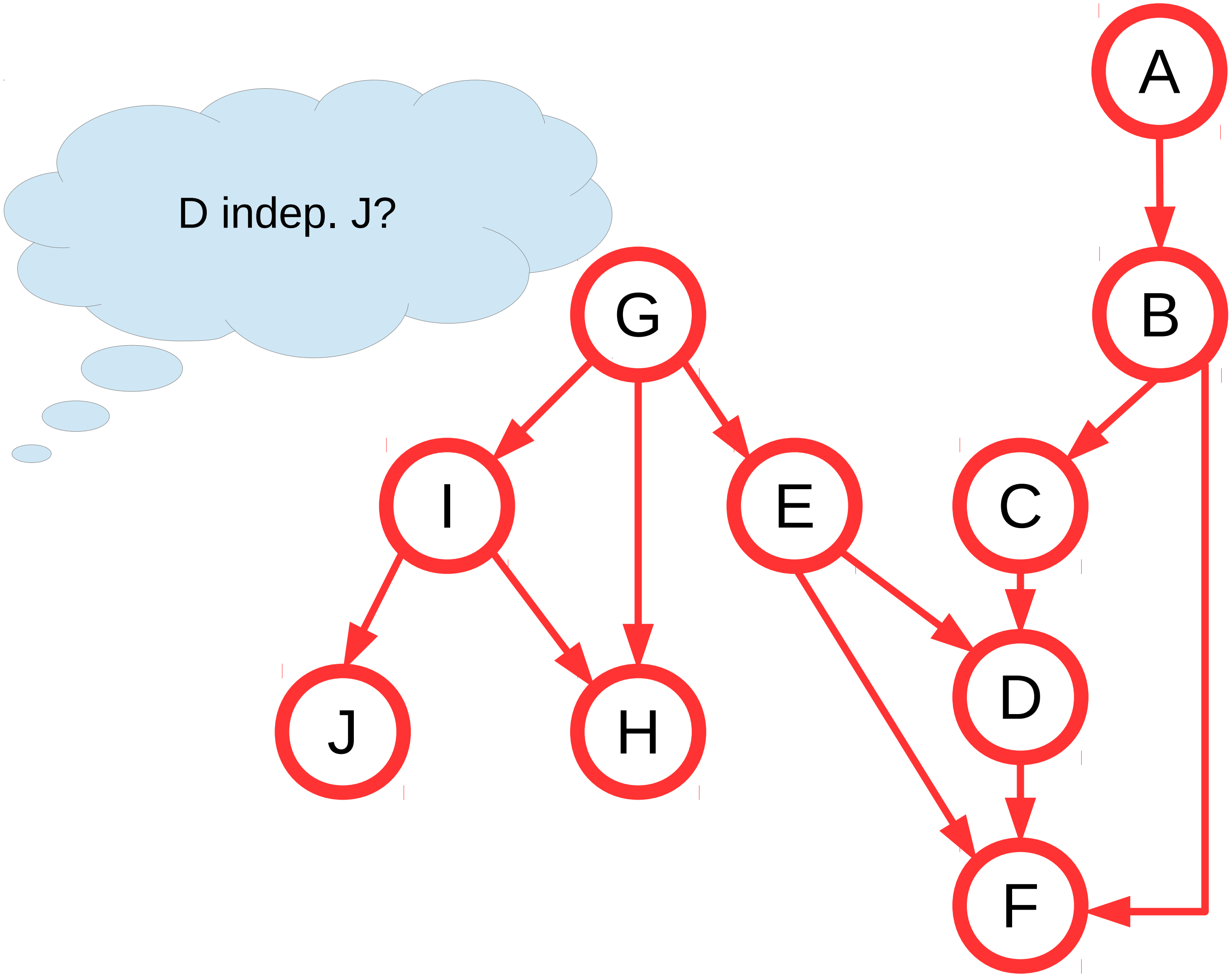
1. Bayesian Networks: d-separation

- Solutions task 1



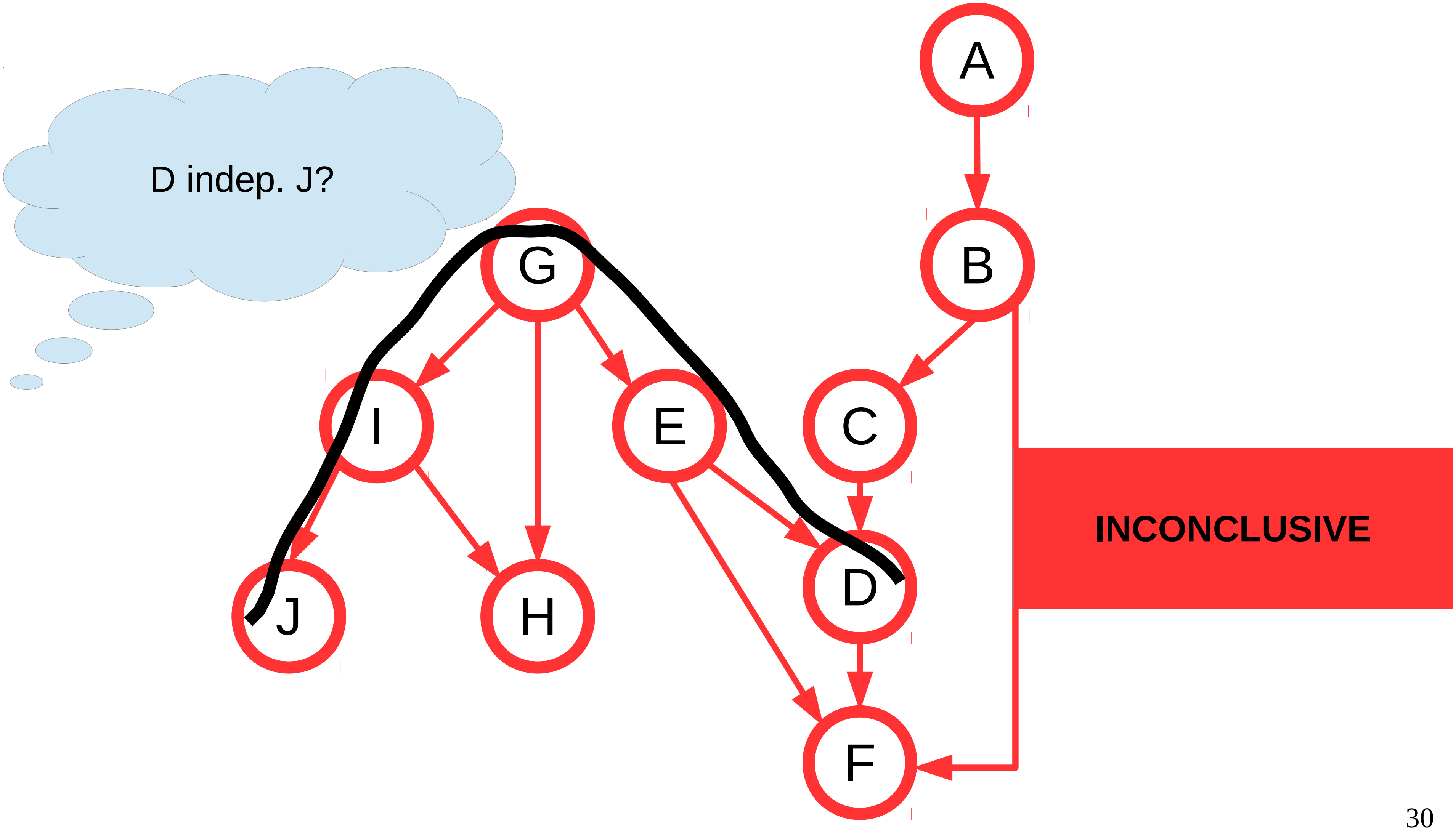
1. Bayesian Networks: d-separation

- Solutions task 1



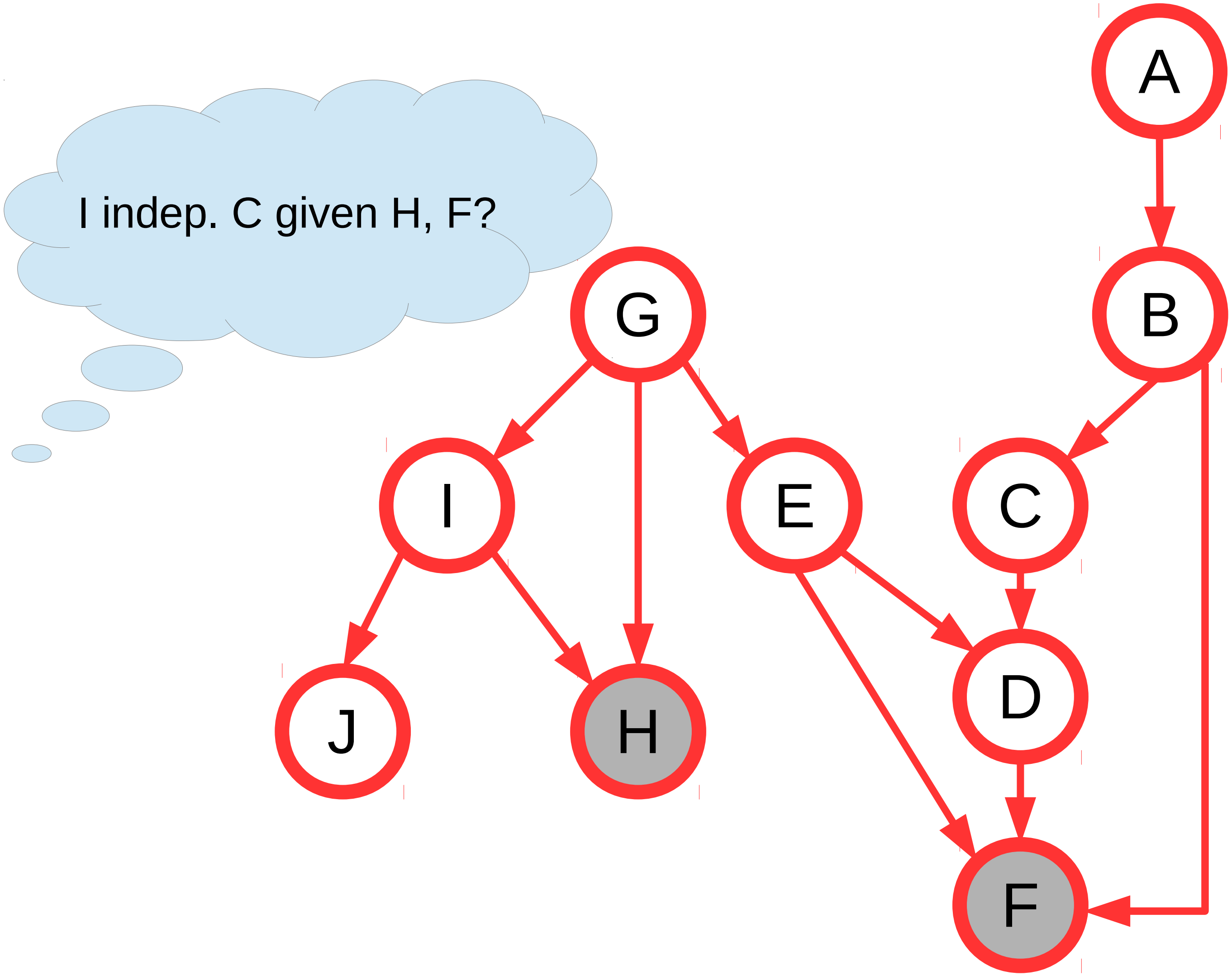
1. Bayesian Networks: d-separation

- Solutions task 1



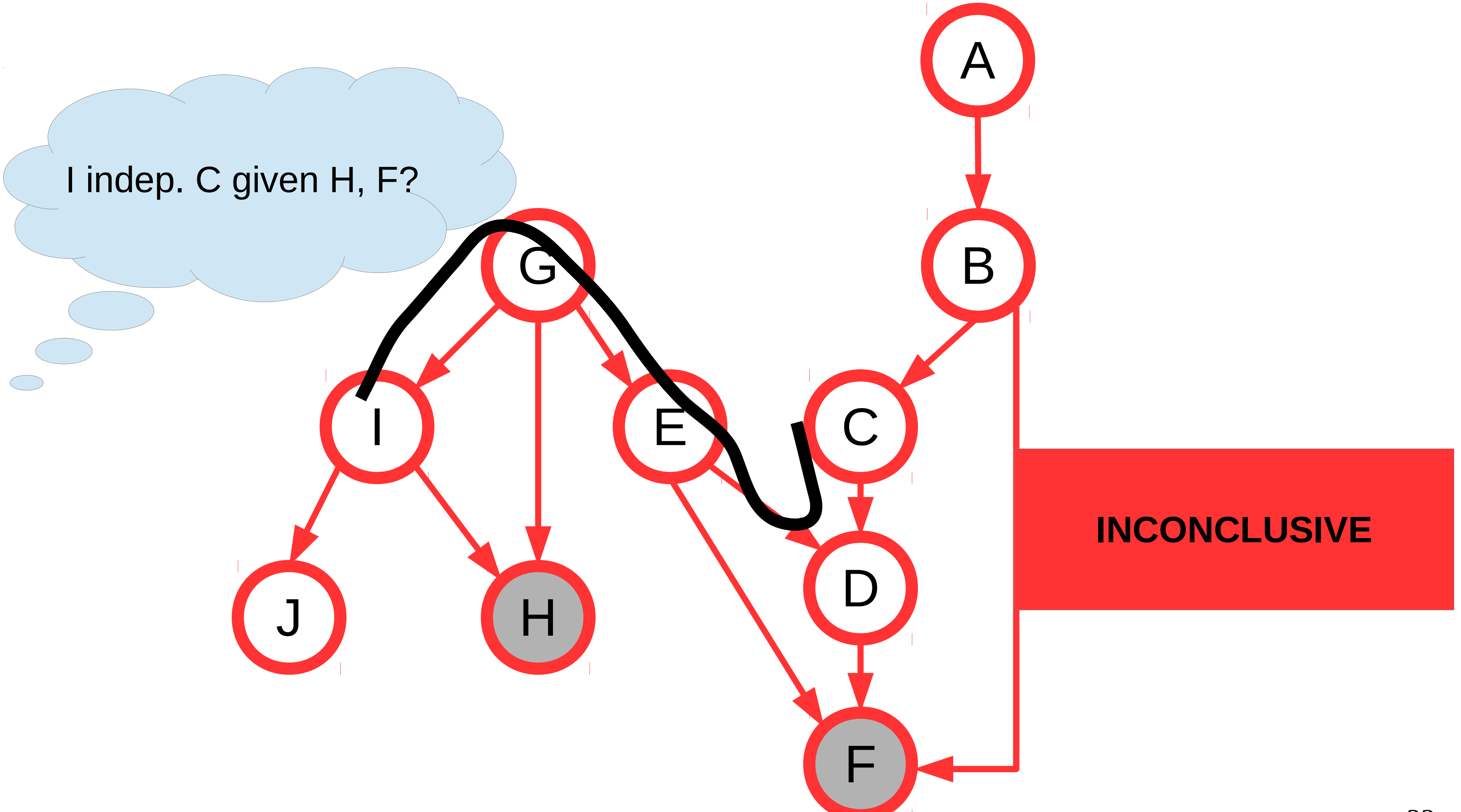
1. Bayesian Networks: d-separation

- Solutions task 1

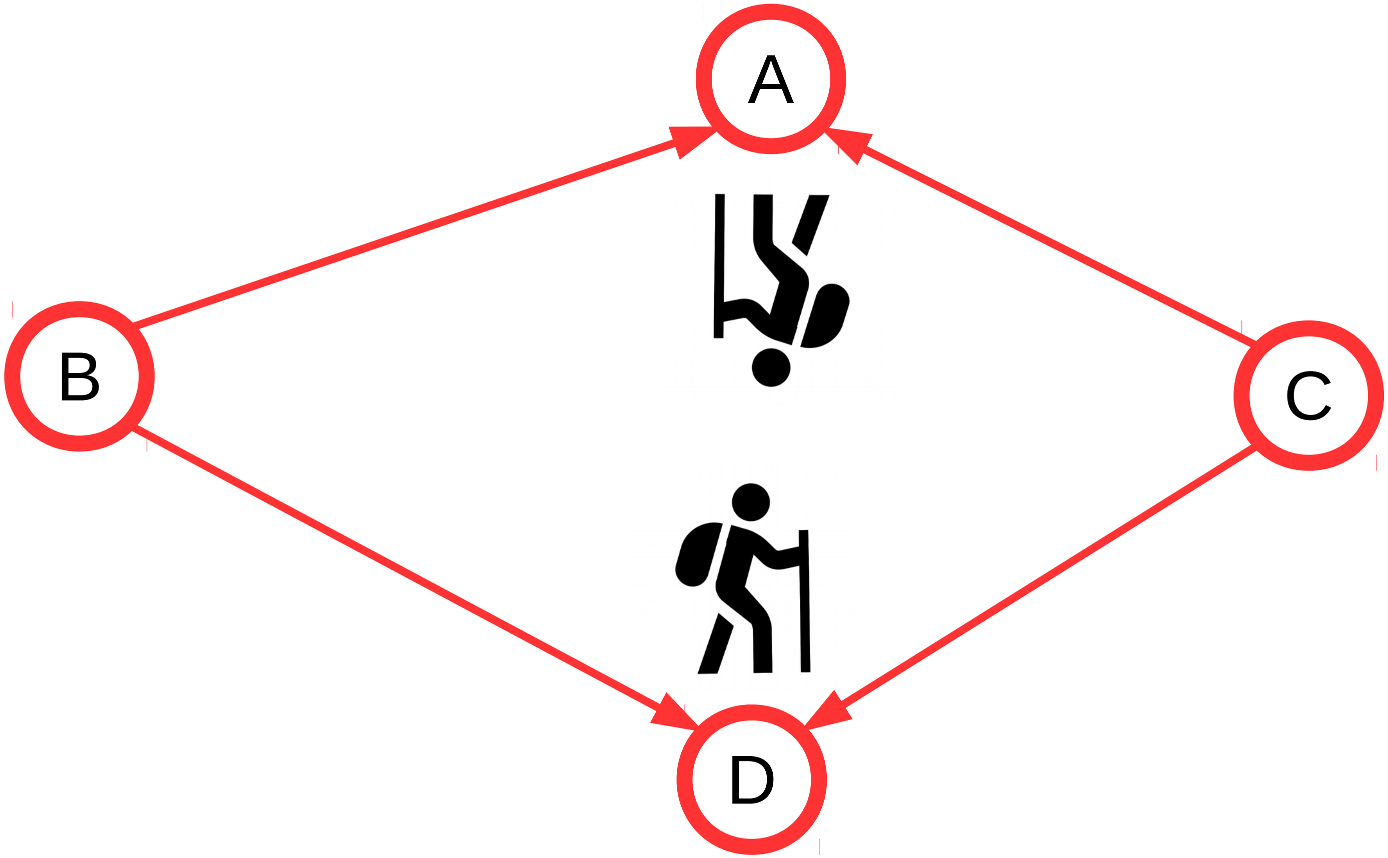


1. Bayesian Networks: d-separation

- Solutions task 1

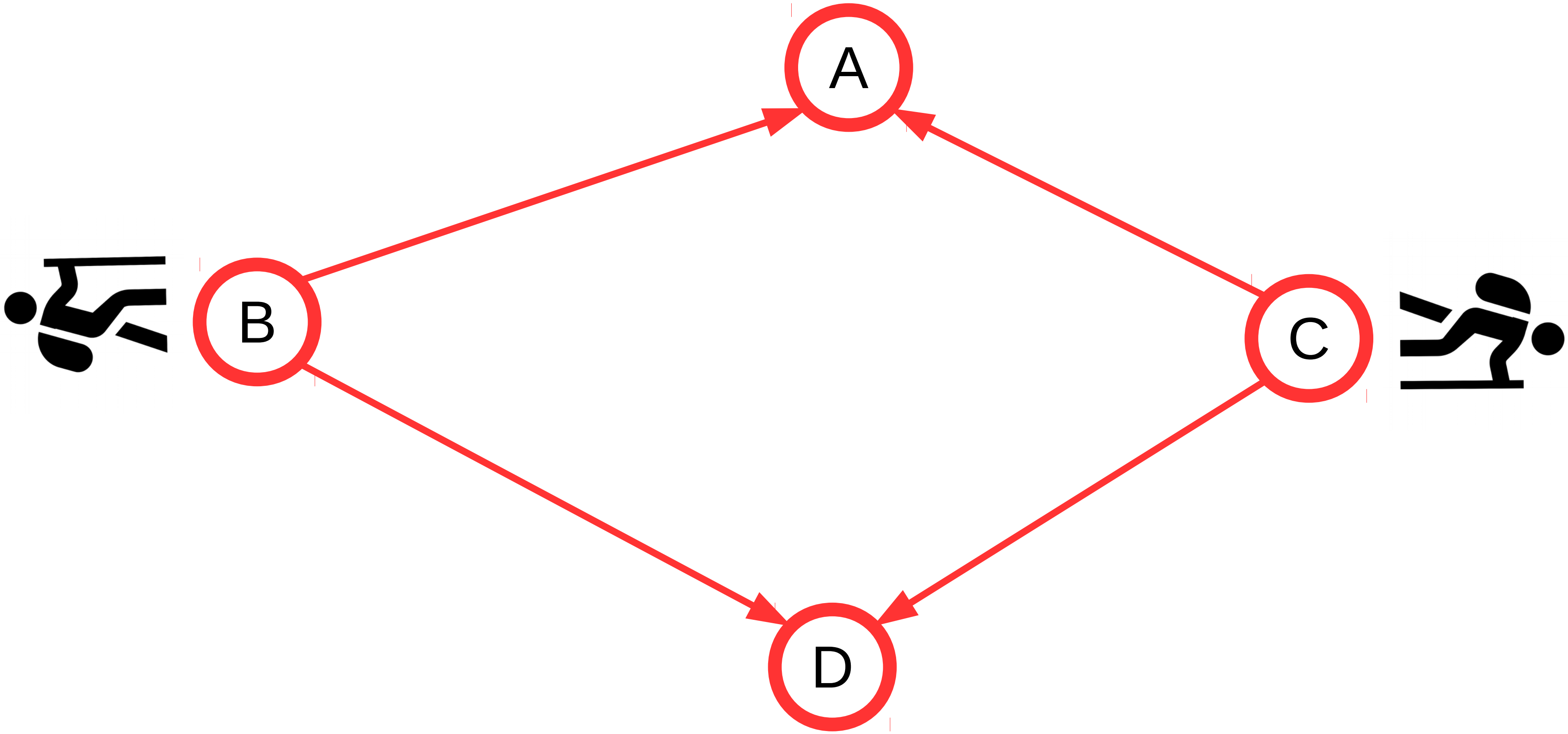


Be careful with the direction of the arrows!



B indep from C ?

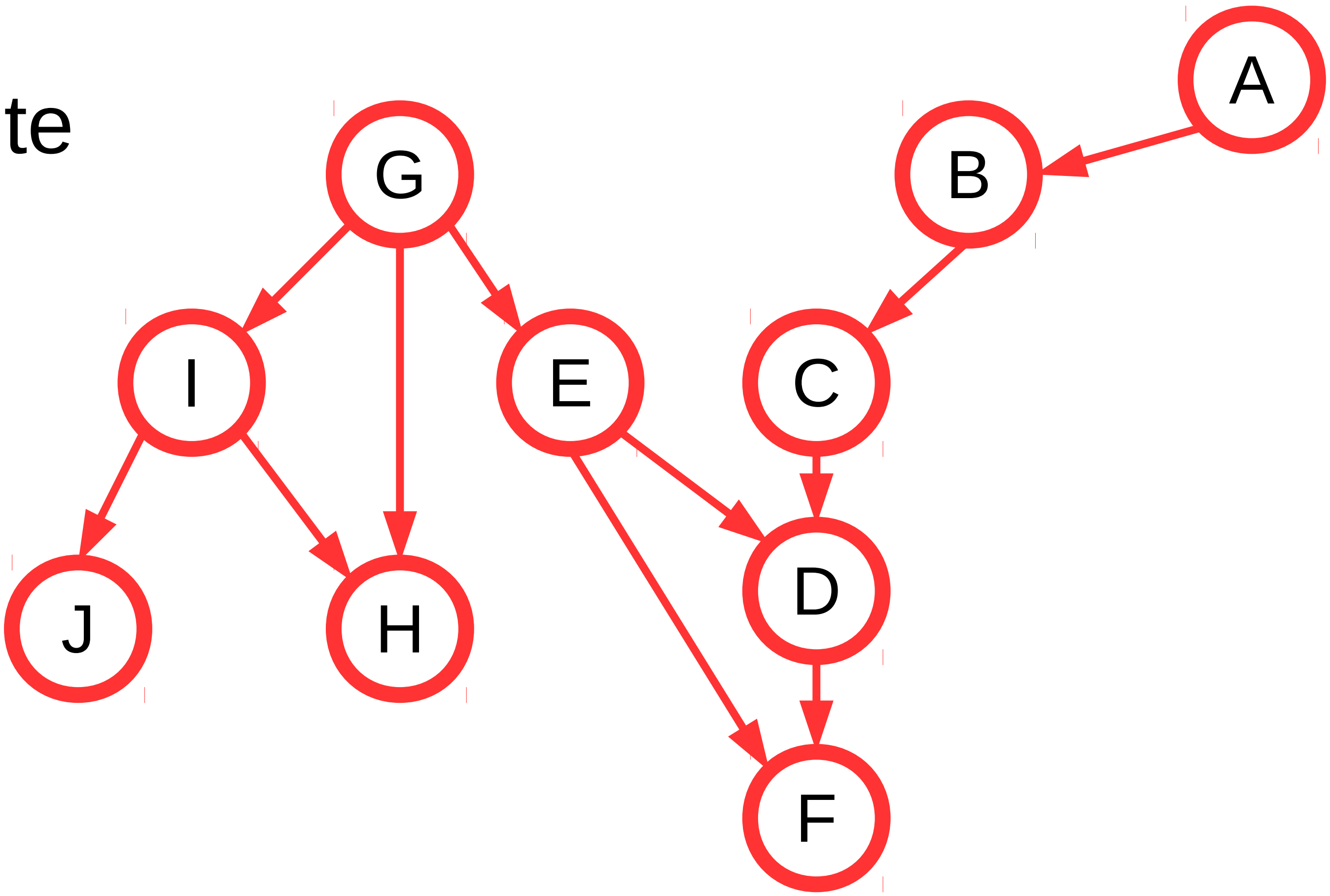
Be careful with the direction of the arrows!



A indep from D ?

2. Variable elimination

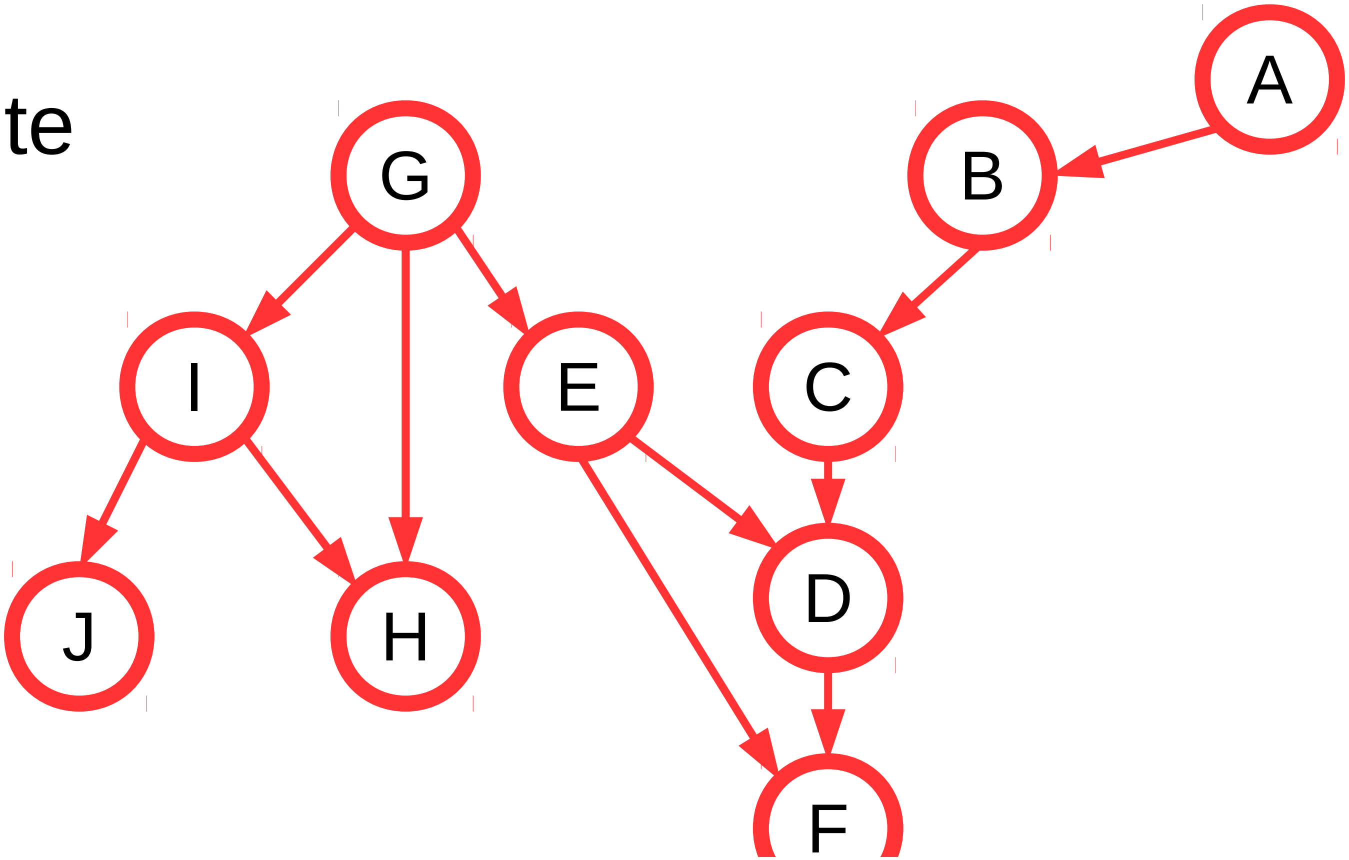
- Compute P(J = j)



1. Joint probability implied by BN structure: $P(A,\ldots,J) = P(A) \cdot P(B|A) \cdot P(C|B) \cdot P(G) \cdot P(E|G) \cdot P(D|C,E) \cdot P(F|B,E,D) \cdot P(I|G) \cdot P(H|G,I) \cdot P(J|I)$

2. Variable elimination

- Compute P(J = j)



2. Eliminating A:

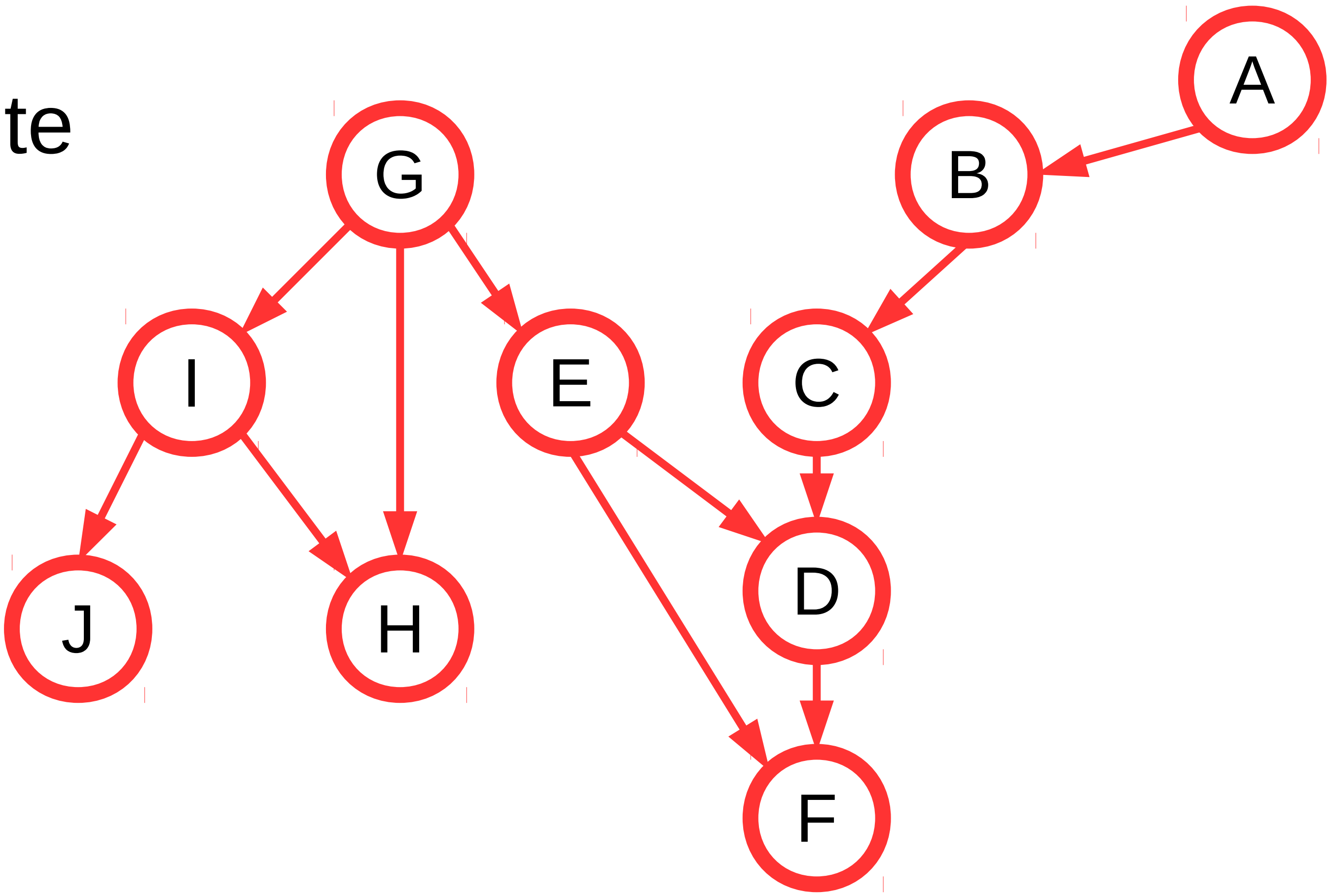
$$P(B,\dots,J) = \sum_a P(a)\cdot P(B|a)\cdot P(C|B)\cdot P(G)\cdot P(E|G)\cdot P(D|C,E)\cdot P(F|B,E,D)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)$$

$$= P(C|B)\cdot P(G)\cdot P(E|G)\cdot P(D|C,E)\cdot P(F|B,E,D)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot \sum_a P(a)P(B|a)$$

$$= P(C|B)\cdot P(G)\cdot P(E|G)\cdot P(D|C,E)\cdot P(F|B,E,D)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_1(B)$$

2. Variable elimination

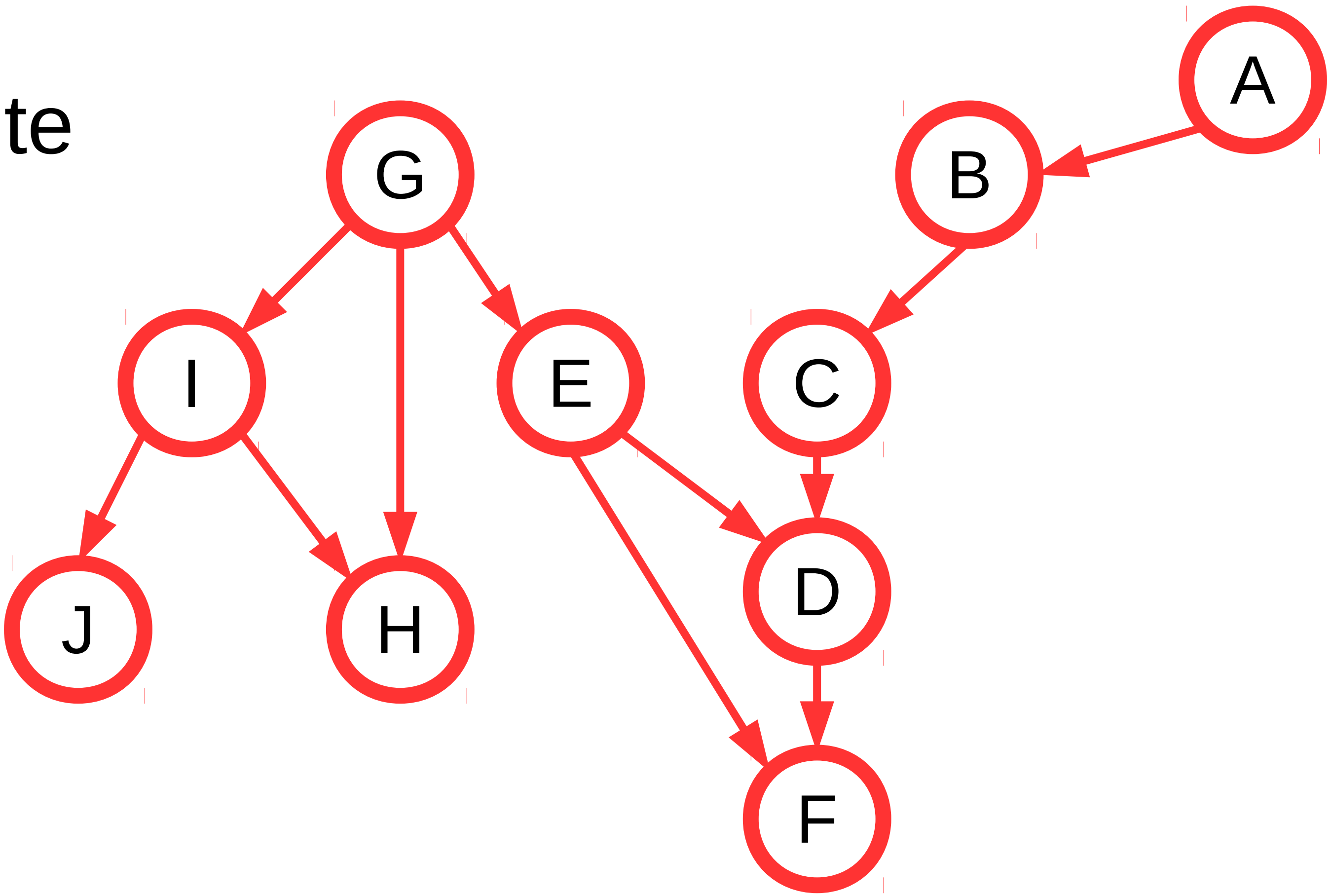
- Compute $P(J = j)$



3. Eliminating B: $P(C, \ldots, J) = P(G) \cdot P(E|G) \cdot P(D|C, E) \cdot P(I|G) \cdot P(H|G, I) \cdot P(J|I) \cdot g_2(C, F, E, D)$

2. Variable elimination

- Compute P(J = j)

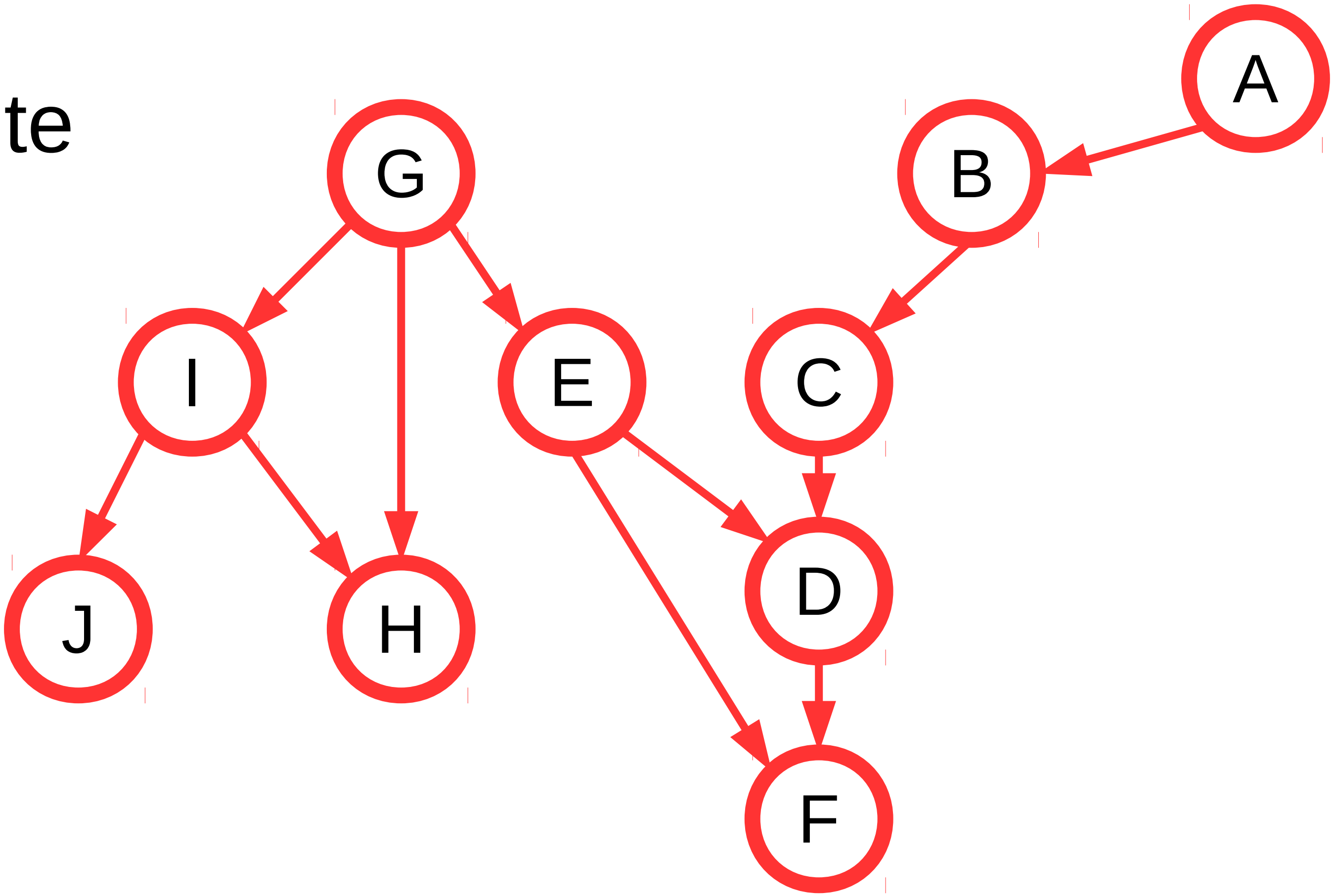


4. Eliminating C: $P(D,\ldots,J) = P(G)\cdot P(E|G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_3(F,E,D)$

5. Eliminating D: $P(E,\ldots,J) = P(G)\cdot P(E|G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_4(F,E)$

2. Variable elimination

- Compute P(J = j)

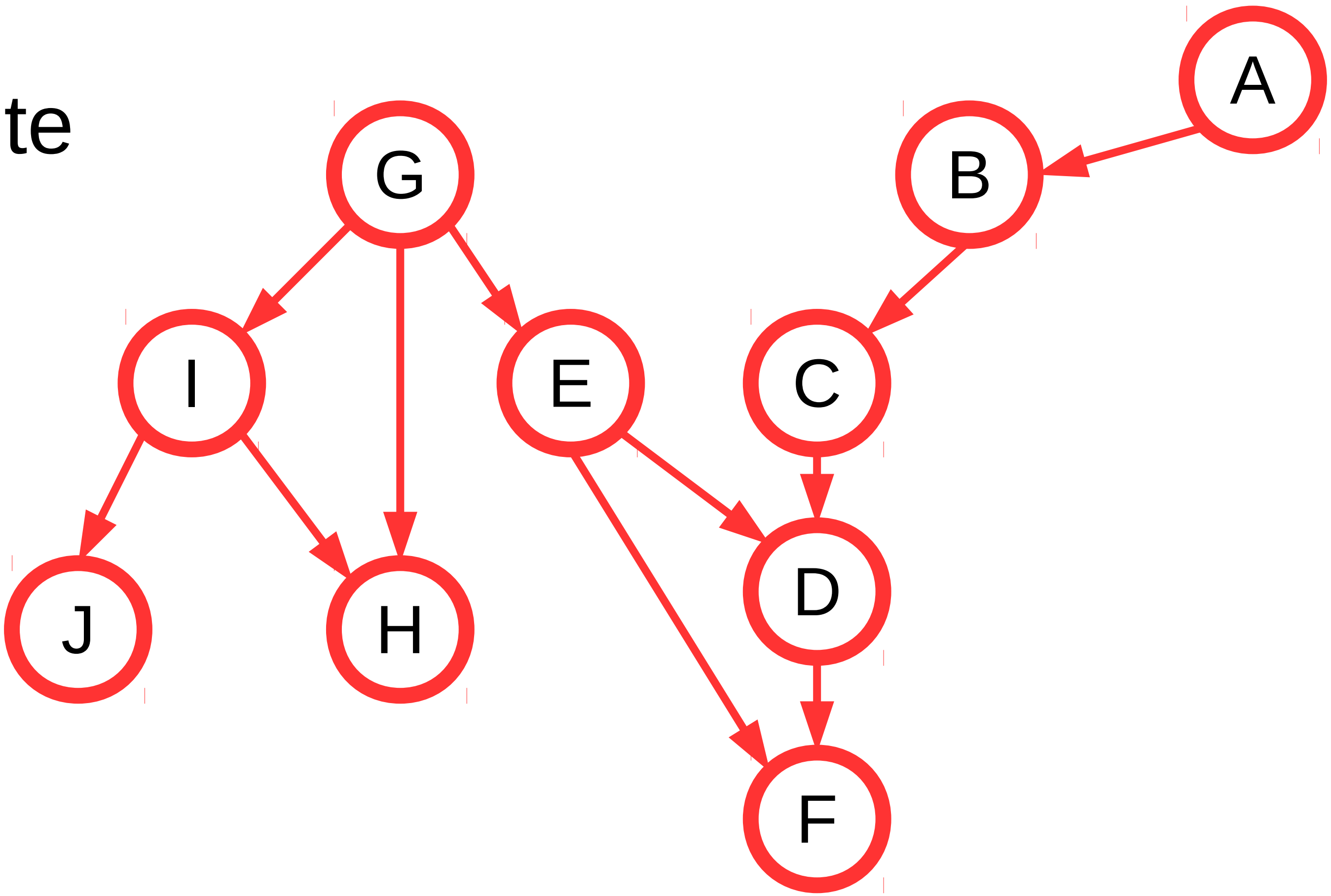


4. Eliminating C: $P(D,\ldots,J) = P(G)\cdot P(E|G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_3(F,E,D)$

5. Eliminating D: $P(E,\ldots,J) = P(G)\cdot P(E|G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_4(F,E)$

2. Variable elimination

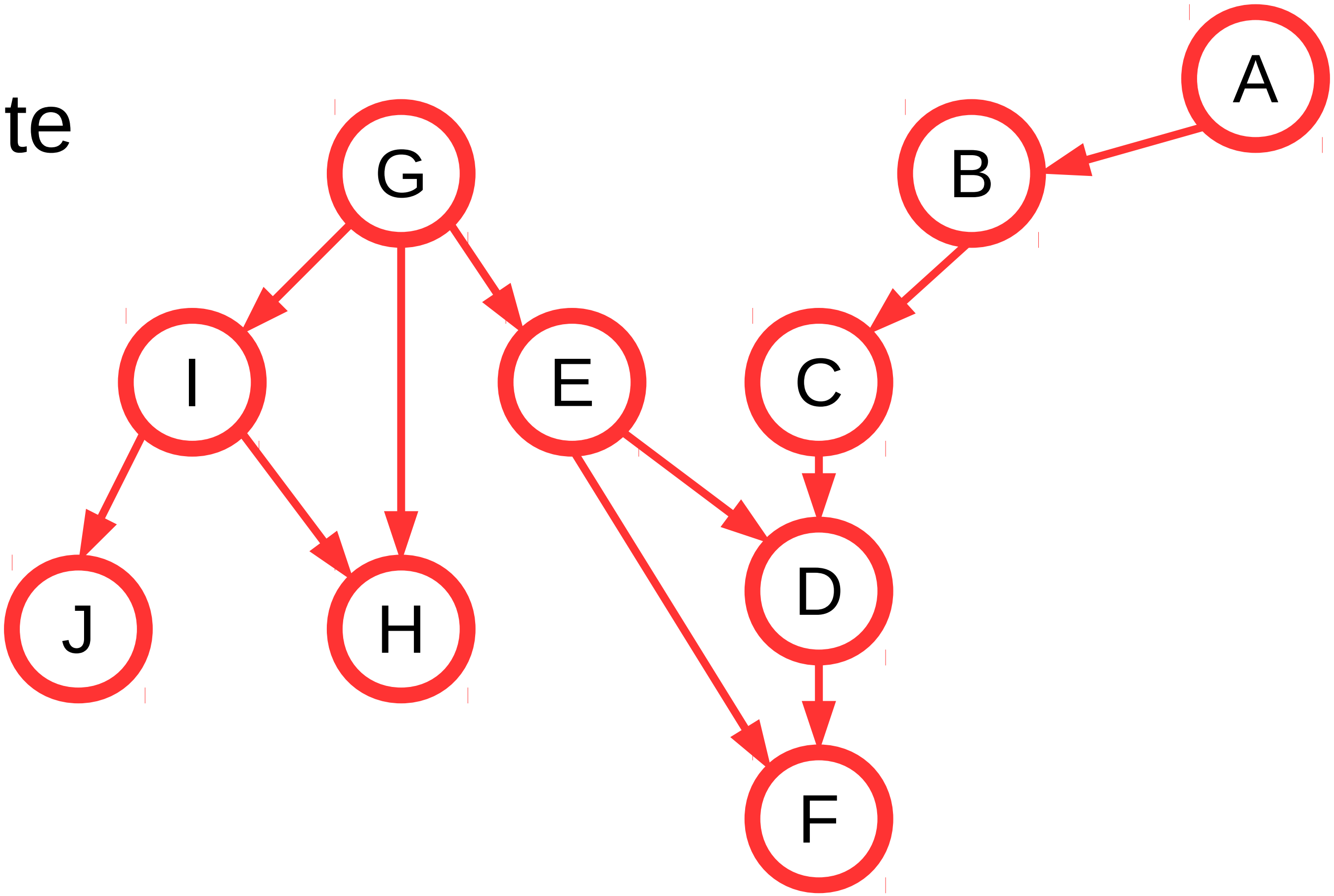
- Compute P(J = j)



6. Eliminating E: $P(F,\ldots,J) = P(G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_5(F,G)$
7. Eliminating F: $P(G,\ldots,J) = P(G)\cdot P(I|G)\cdot P(H|G,I)\cdot P(J|I)\cdot g_6(G)$
8. Eliminating G: $P(H,I,J) = P(J|I)\cdot g_7(I,H)$

2. Variable elimination

- Compute P(J = j)



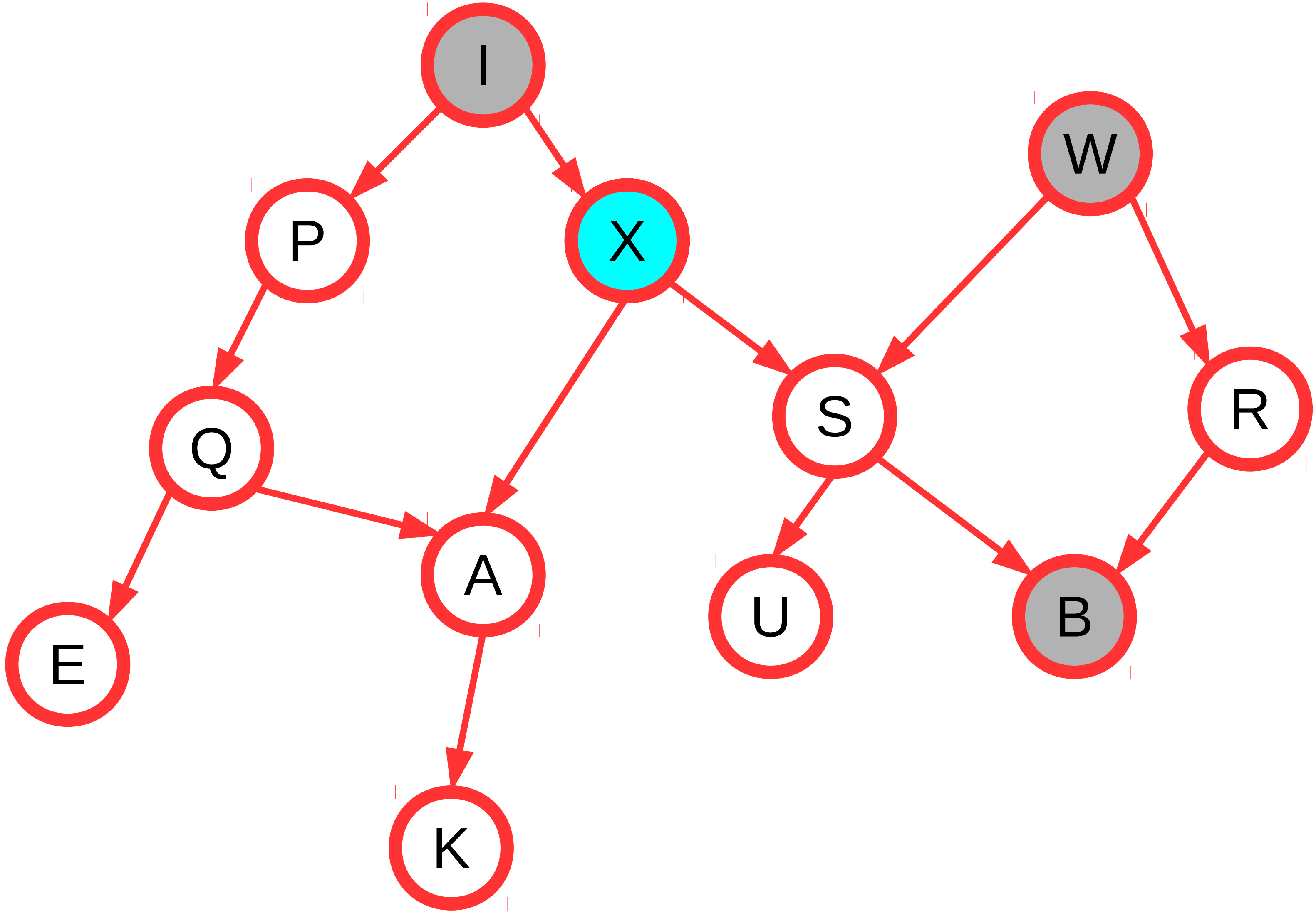
9. Eliminating H: $P(I, J) = P(J|I) \cdot g_8(I)$

10. Eliminating I: $P(J) = g_9(J)$

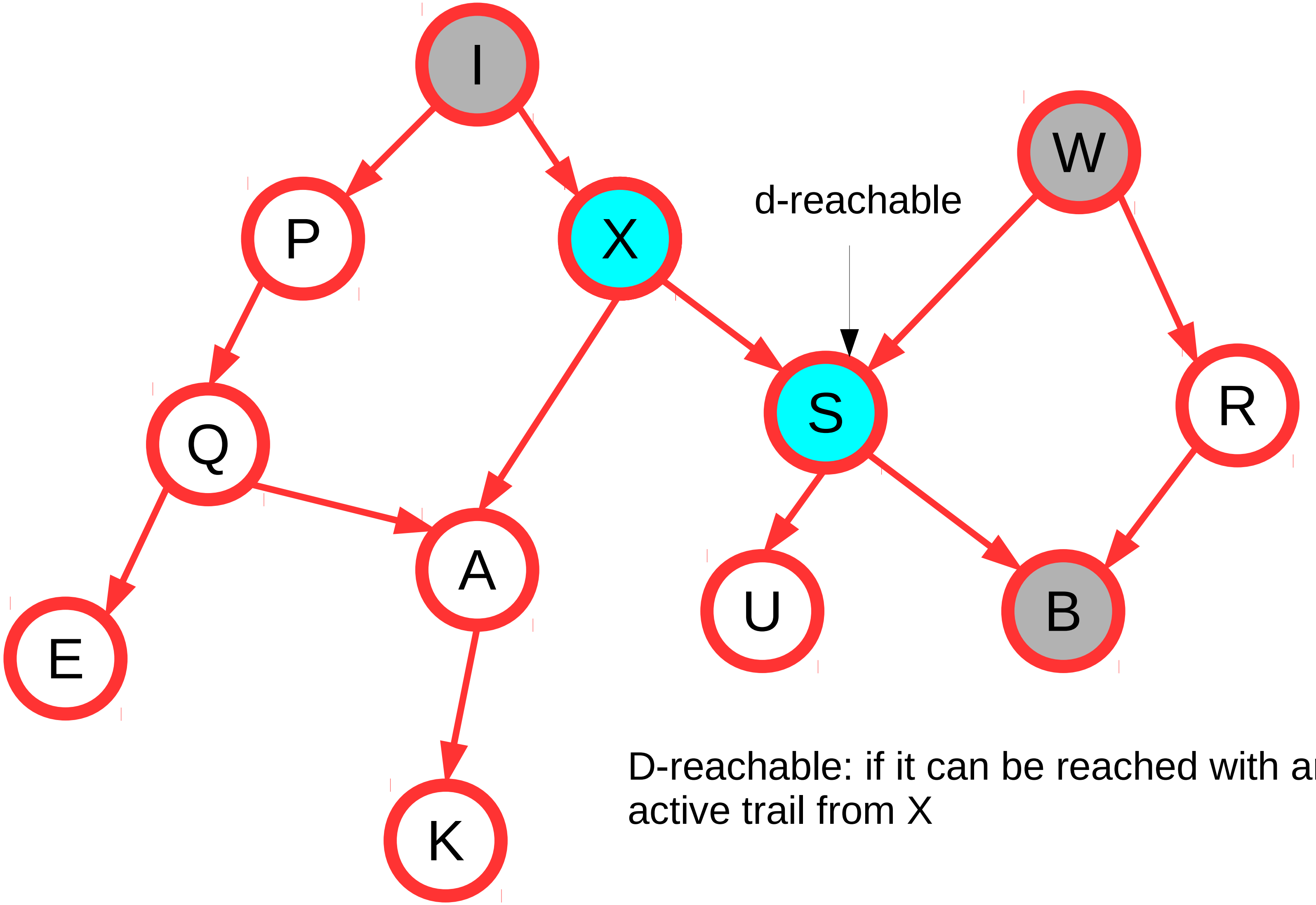
3. An algorithm for d-separation

- Given a Bayesian Network, a variable X, and a set of variables **E** with observed values ***e***, compute all variables Z that are indep. from X given **E.**

3. Algorithm for d-separation

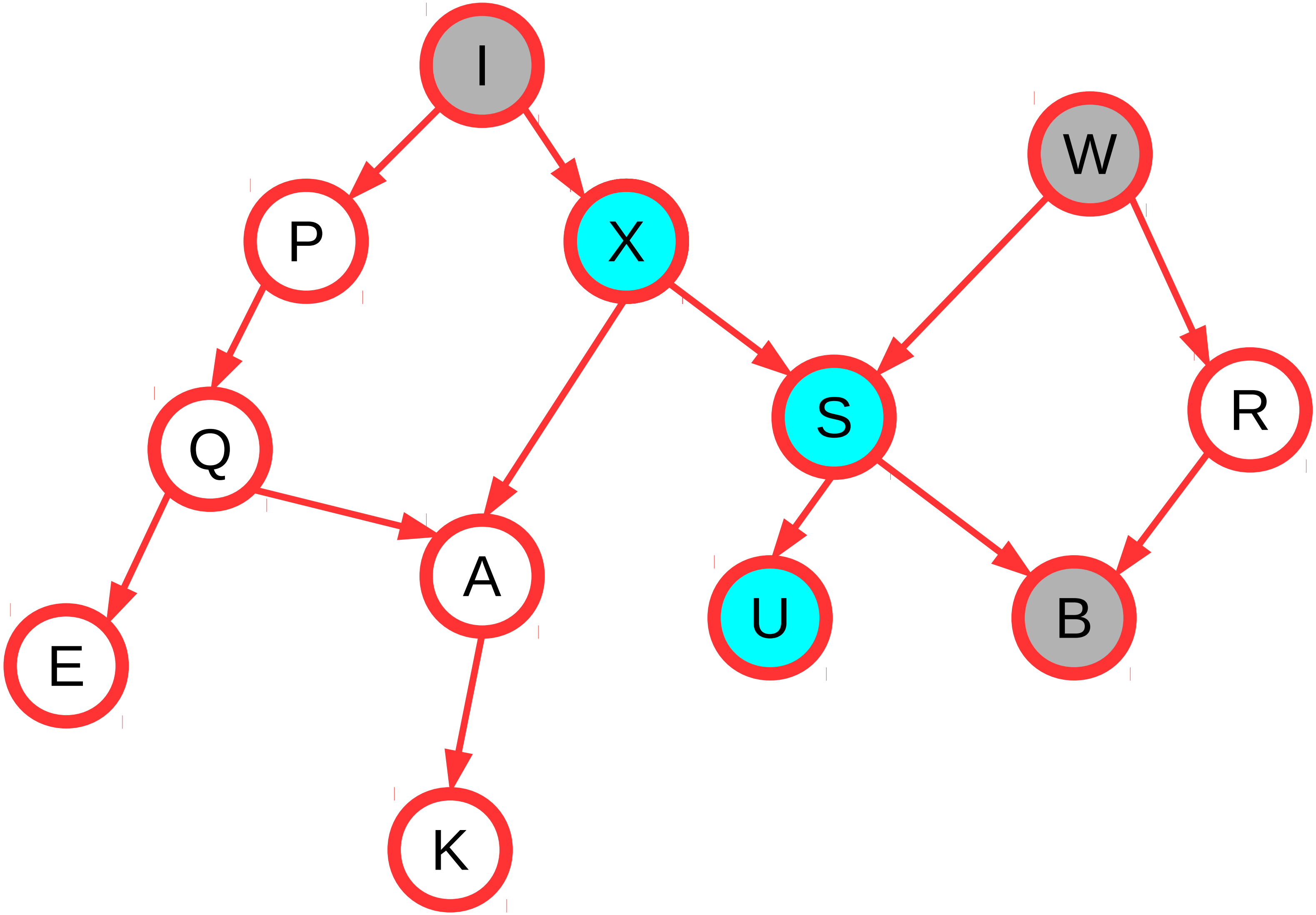


3. Algorithm for d-separation

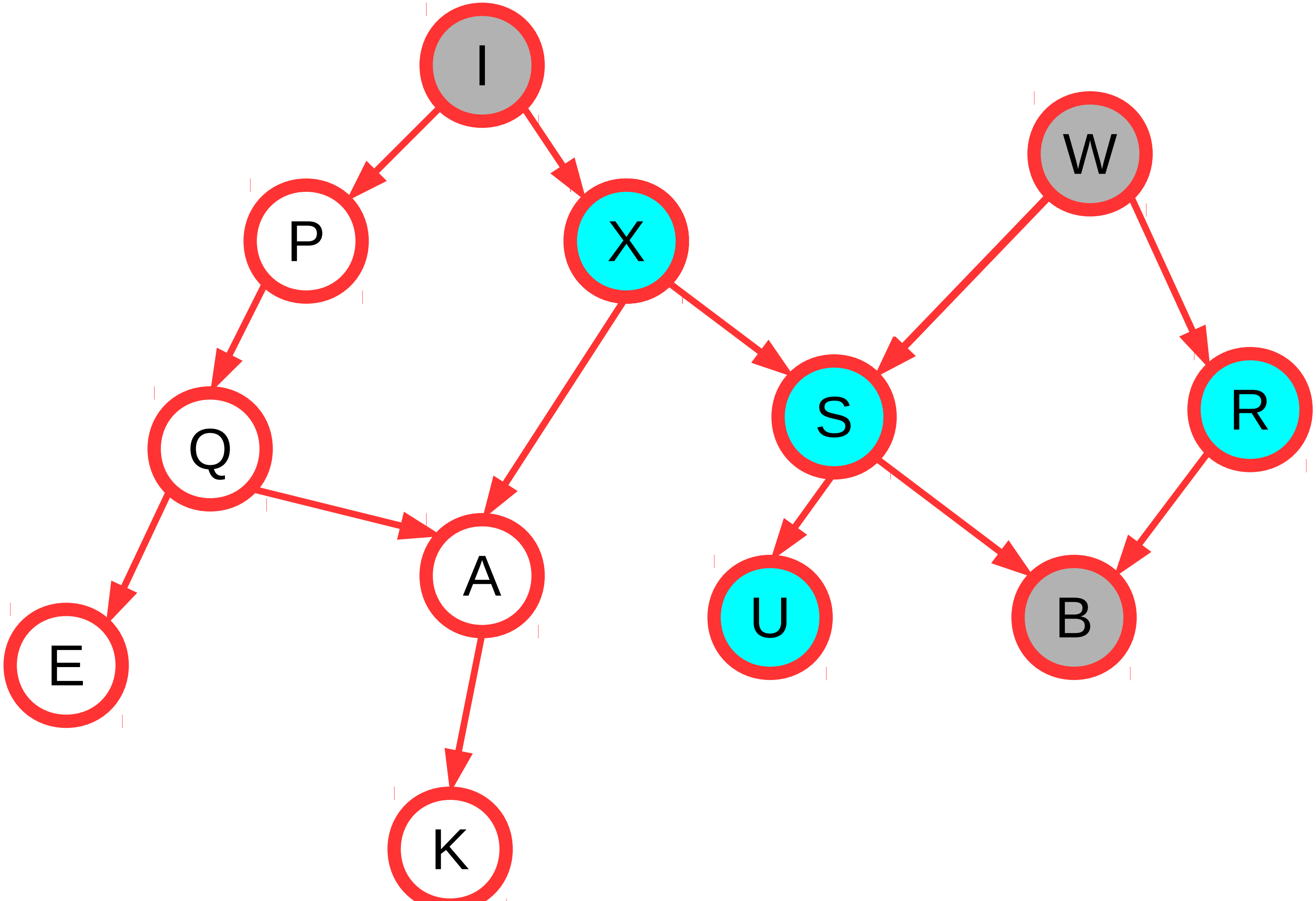


D-reachable: if it can be reached with an active trail from X

3. Algorithm for d-separation



3. Algorithm for d-separation



3. Algorithm for d-separation

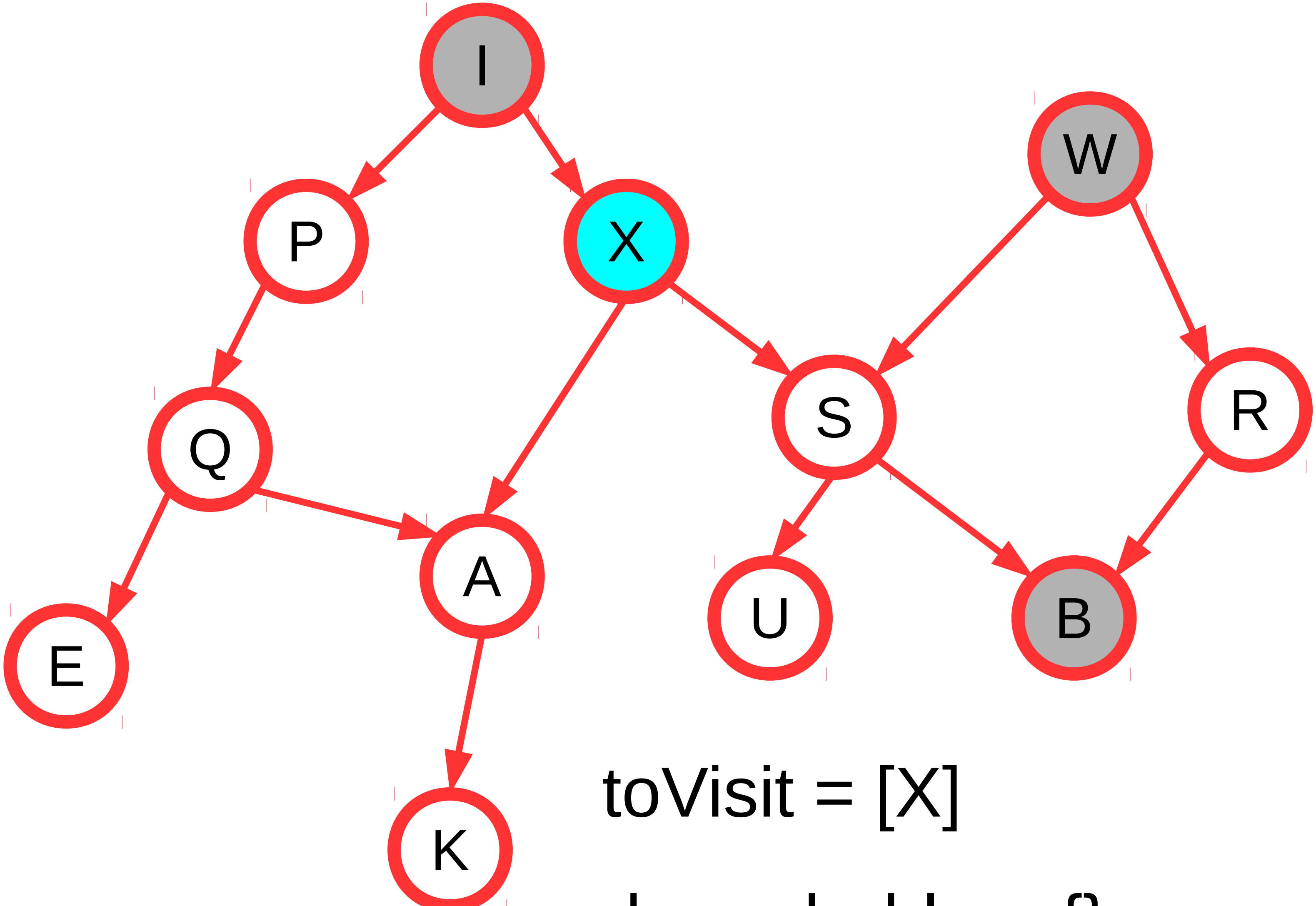
Key insight: Subtrails of active trails are also active!

3. Algorithm for d-separation

Key insight: Subtrails of active trails are also active!

Compute d-reachable variables with a depth-first (or breadth-first) search.

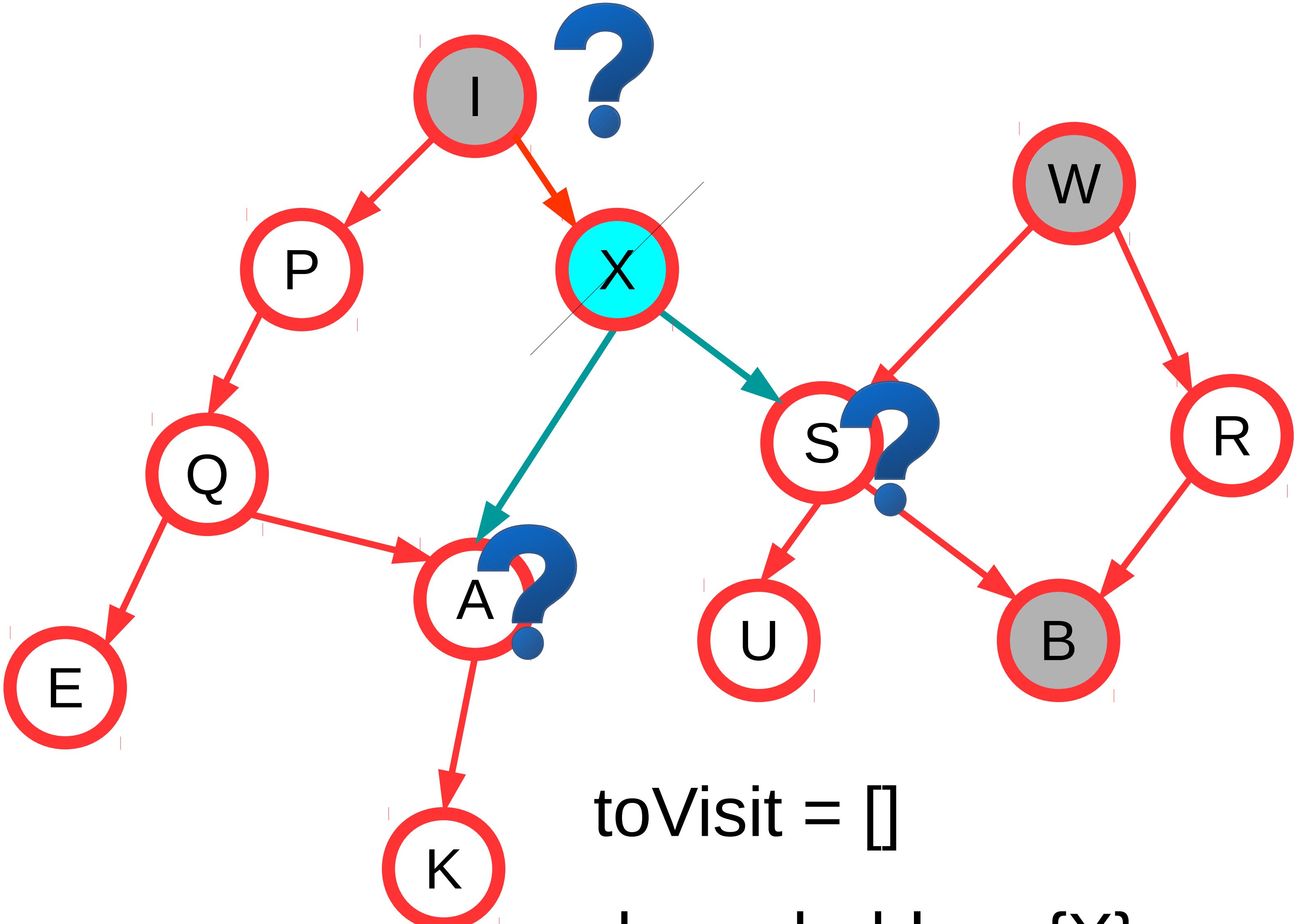
3. Algorithm for d-separation



toVisit = [X]

d-reachable = {}

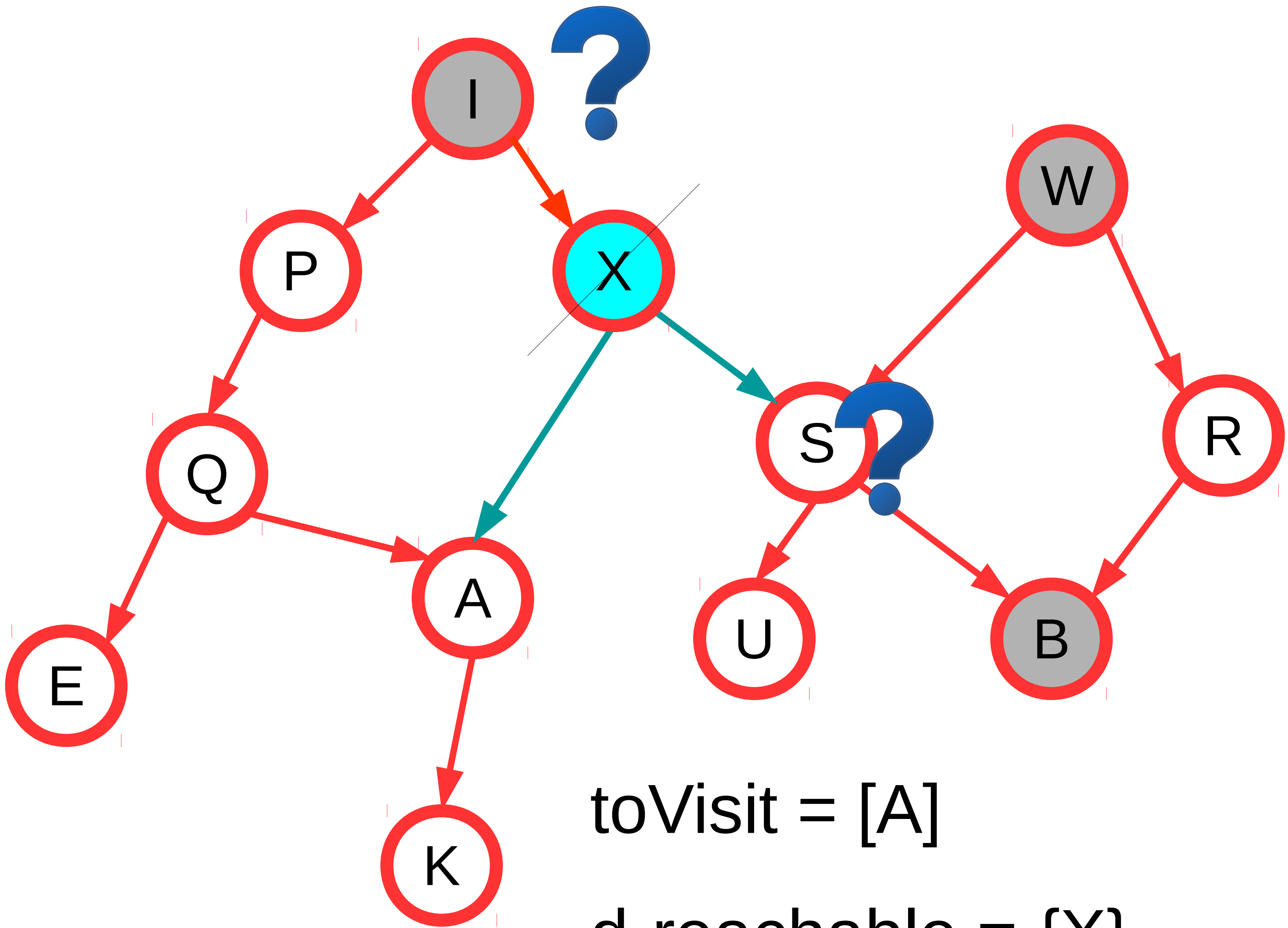
3. Algorithm for d-separation



toVisit = []

d-reachable = {X}

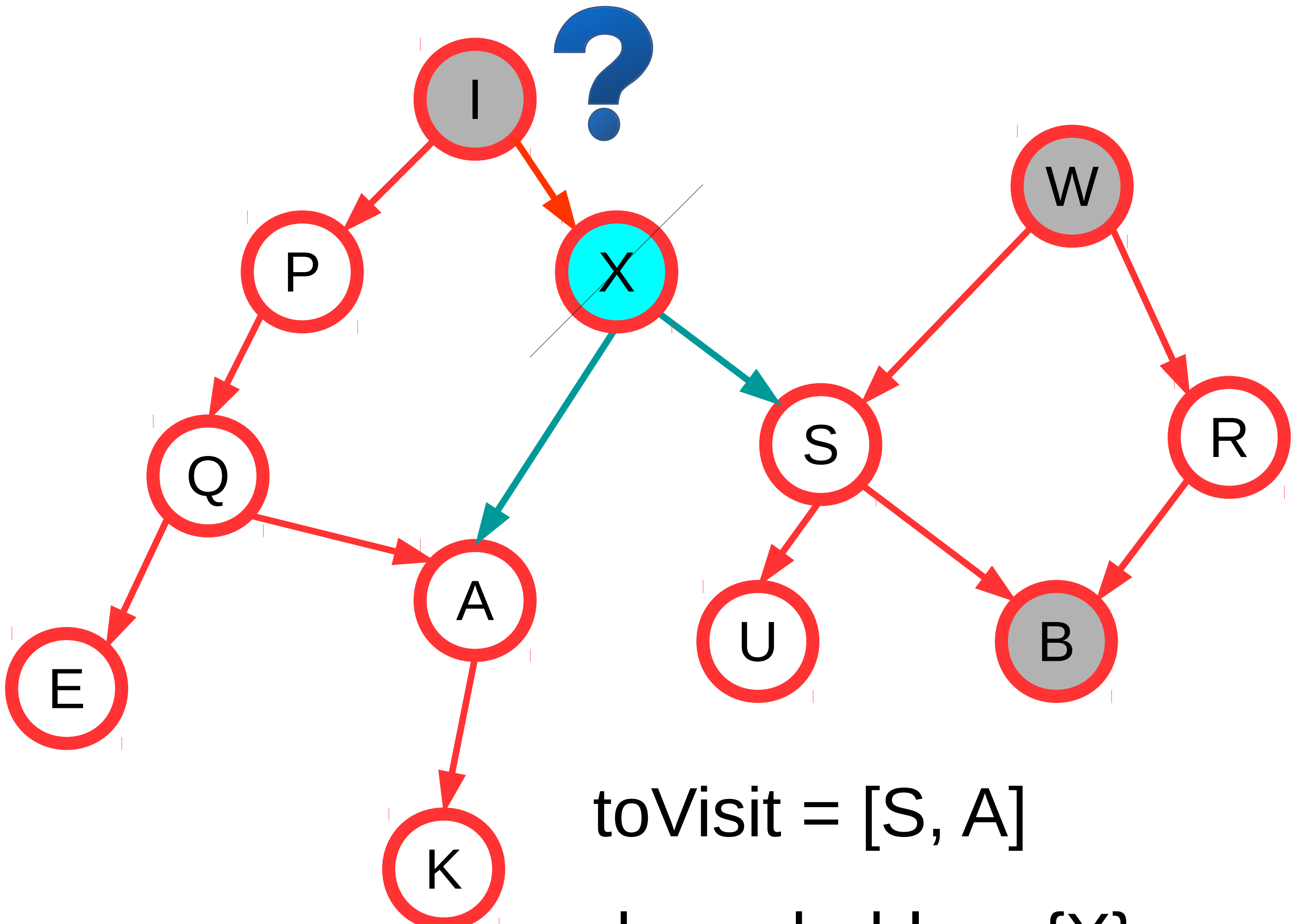
3. Algorithm for d-separation



toVisit = [A]

d-reachable = {X}

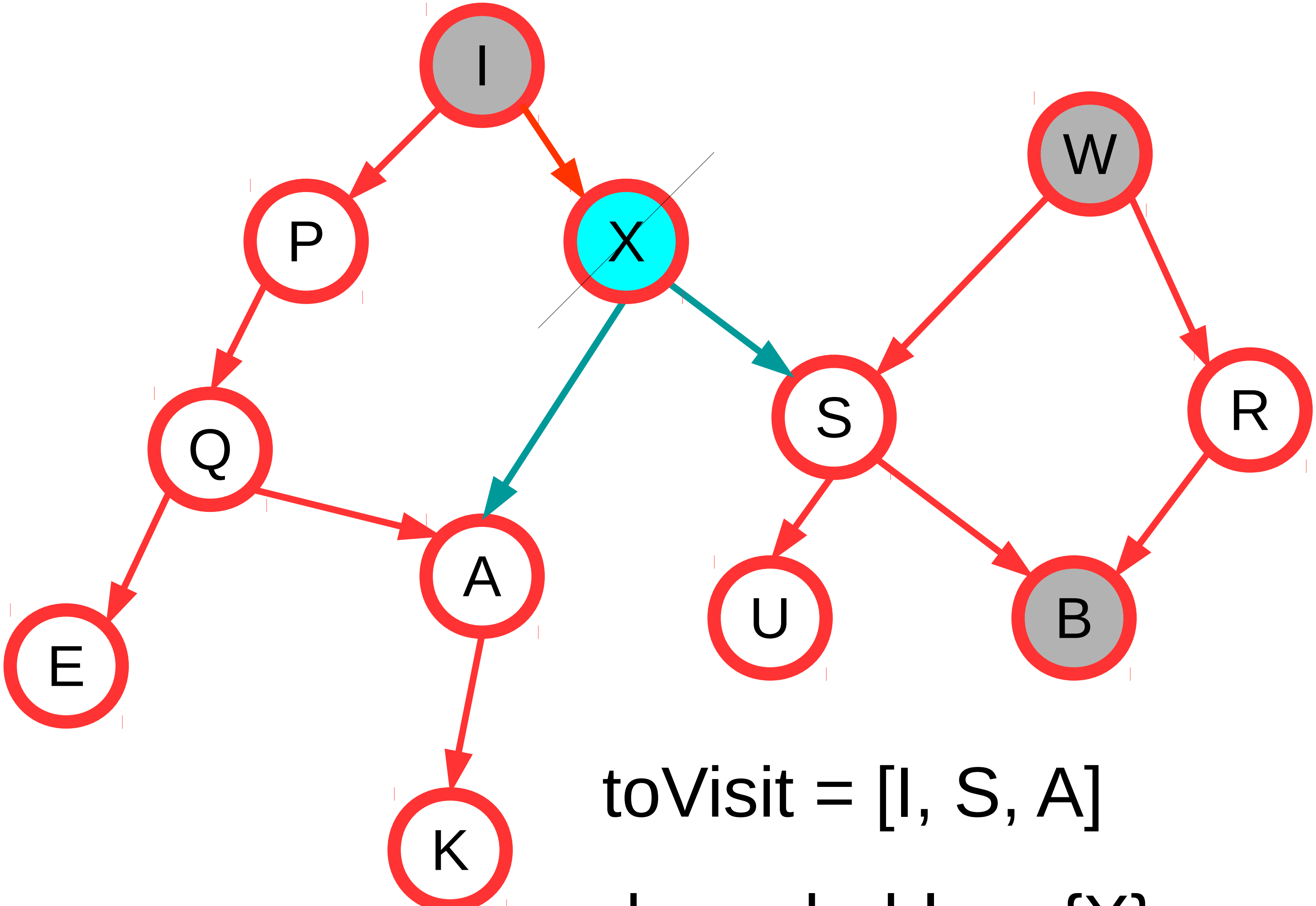
3. Algorithm for d-separation



toVisit = [S, A]

d-reachable = {X}

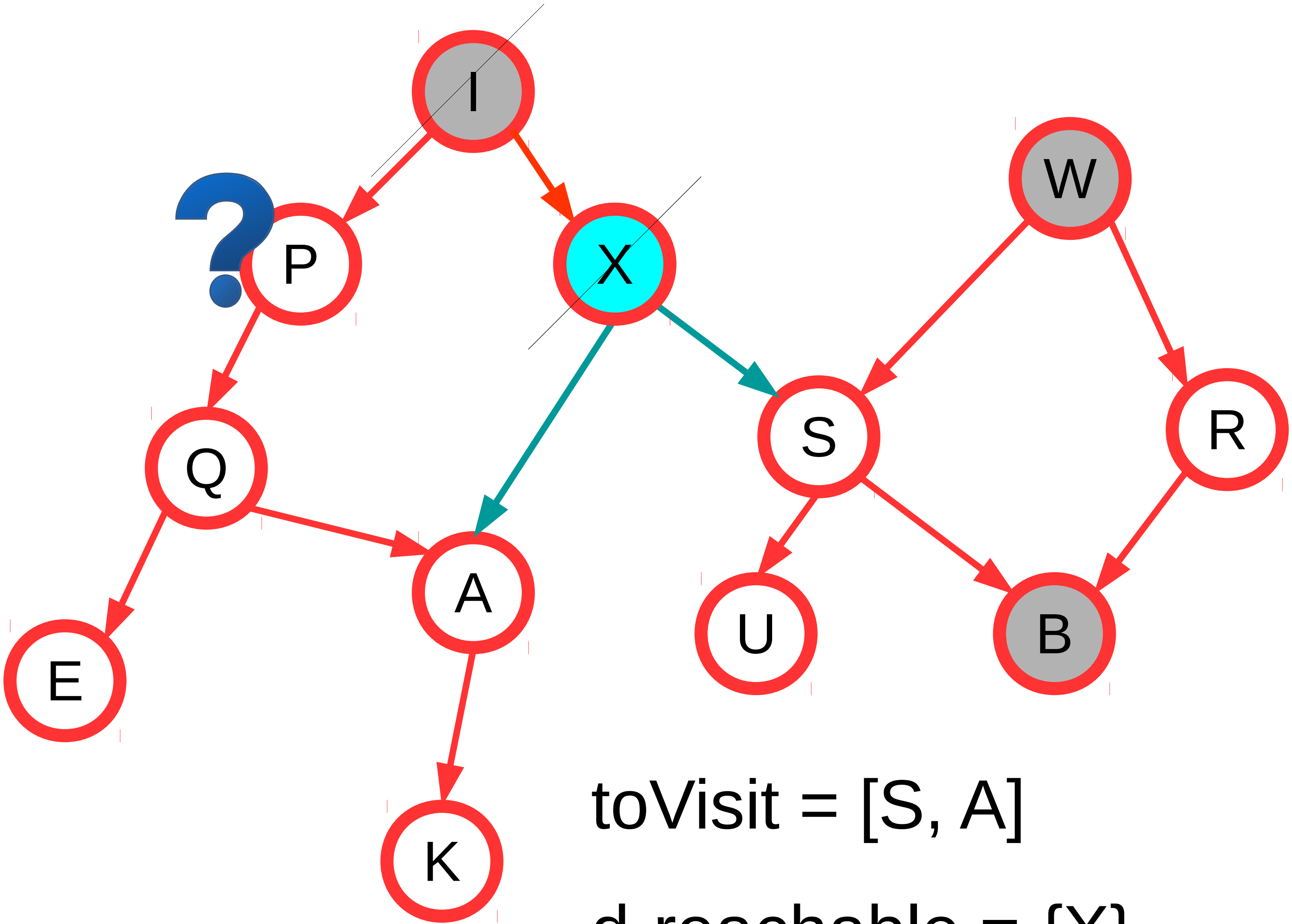
3. Algorithm for d-separation



toVisit = [I, S, A]

d-reachable = {X}

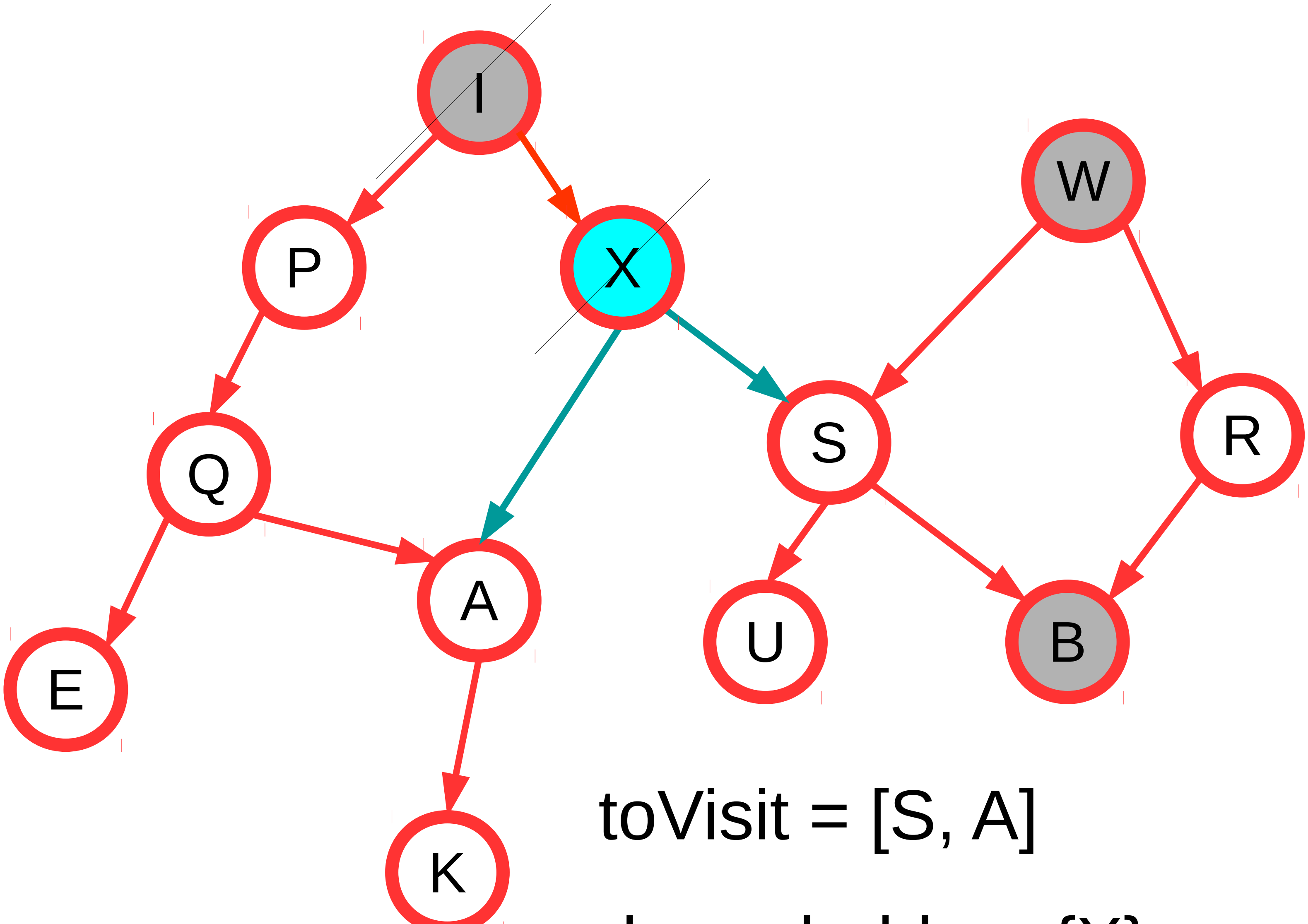
3. Algorithm for d-separation



toVisit = [S, A]

d-reachable = {X}

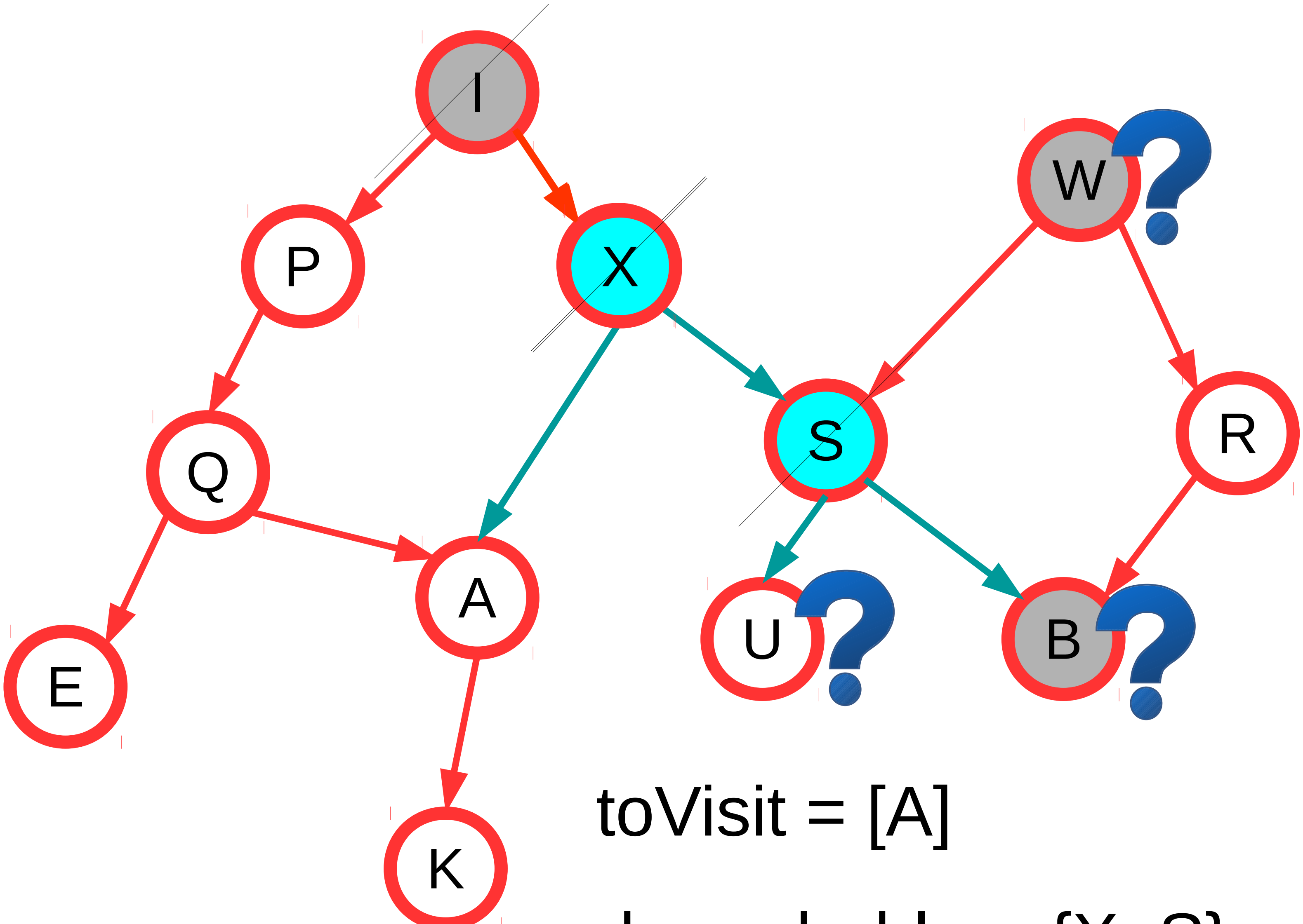
3. Algorithm for d-separation



toVisit = [S, A]

d-reachable = {X}

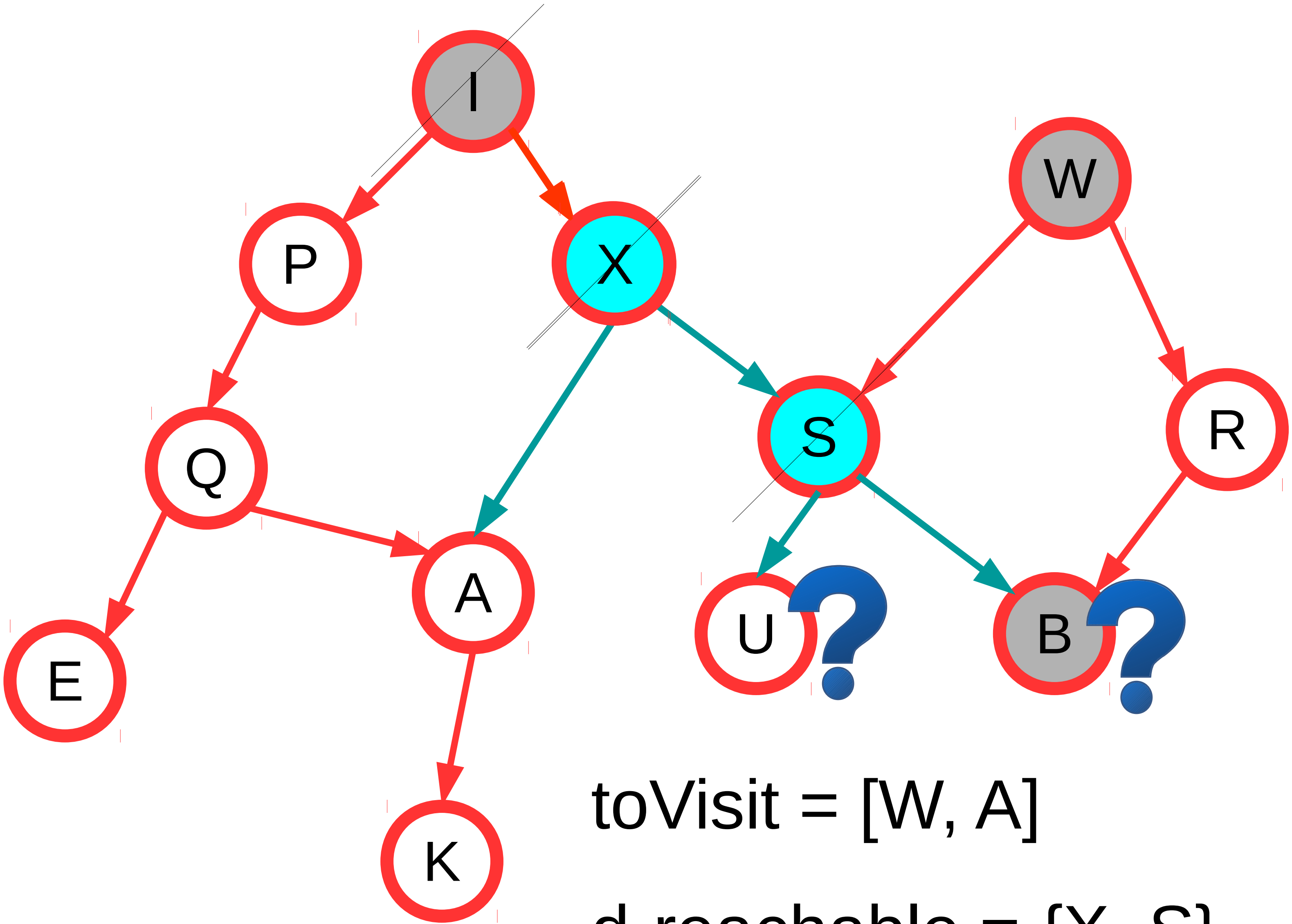
3. Algorithm for d-separation



toVisit = [A]

d-reachable = {X, S}

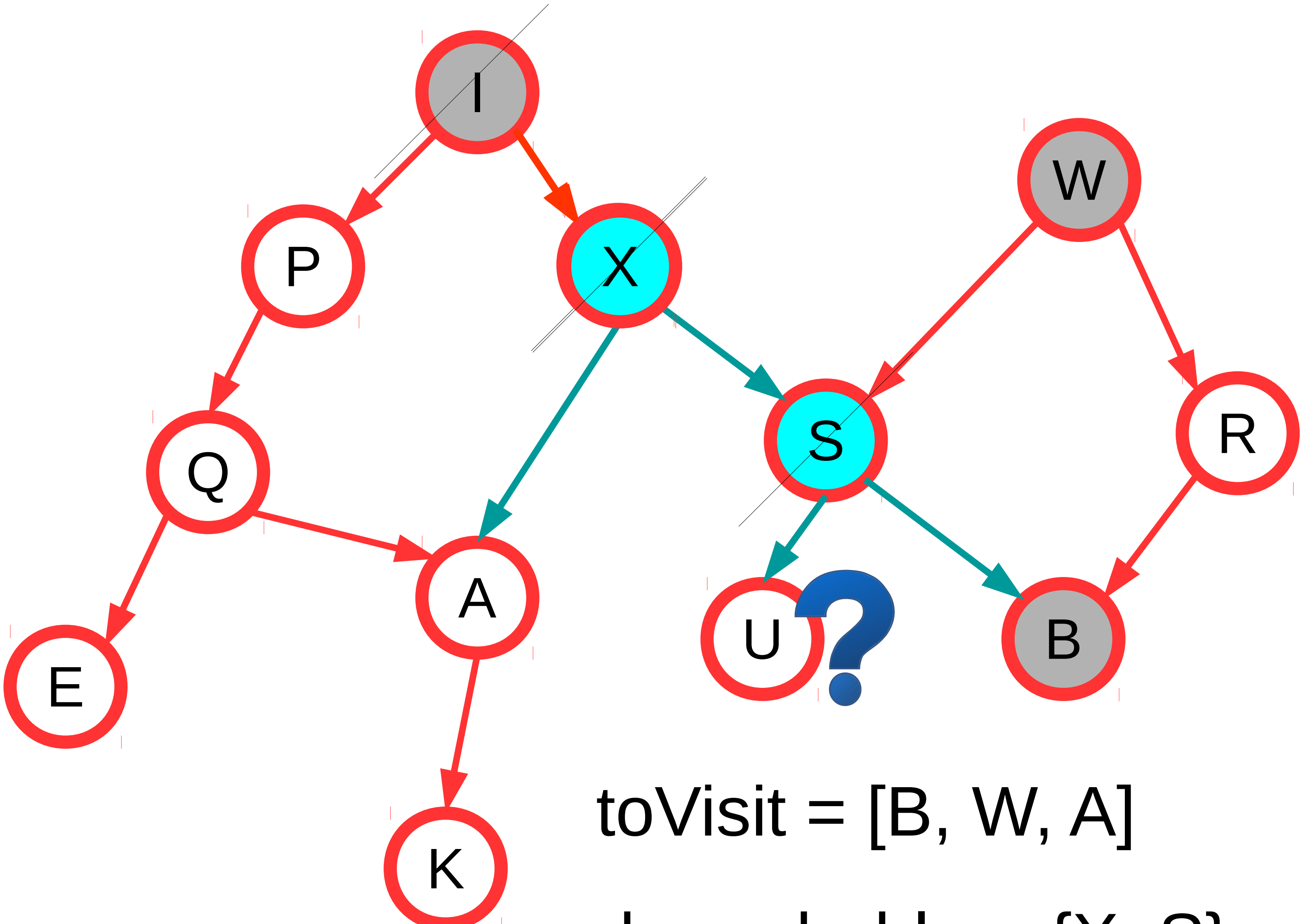
3. Algorithm for d-separation



toVisit = [W, A]

d-reachable = {X, S}

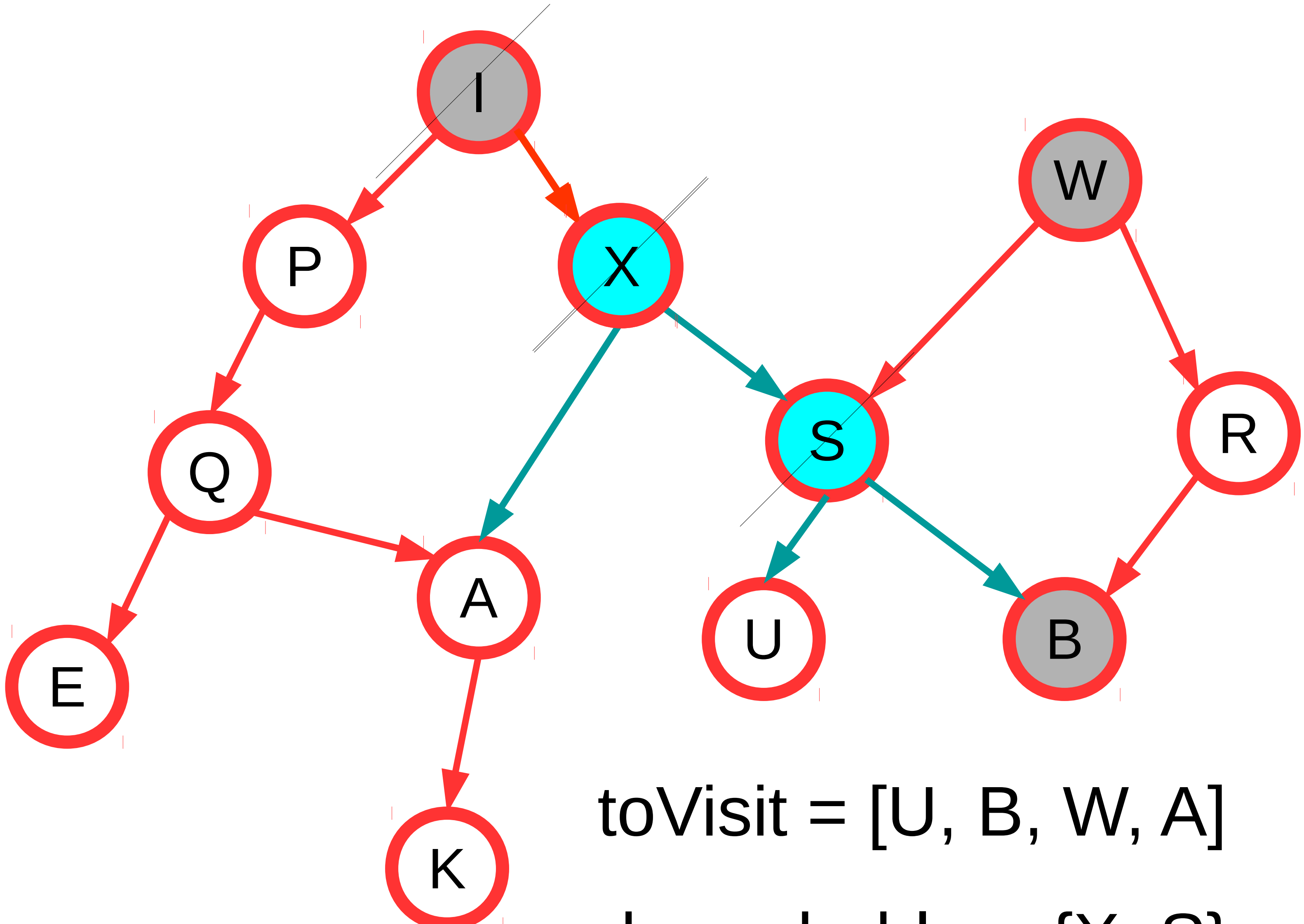
3. Algorithm for d-separation



toVisit = [B, W, A]

d-reachable = {X, S}

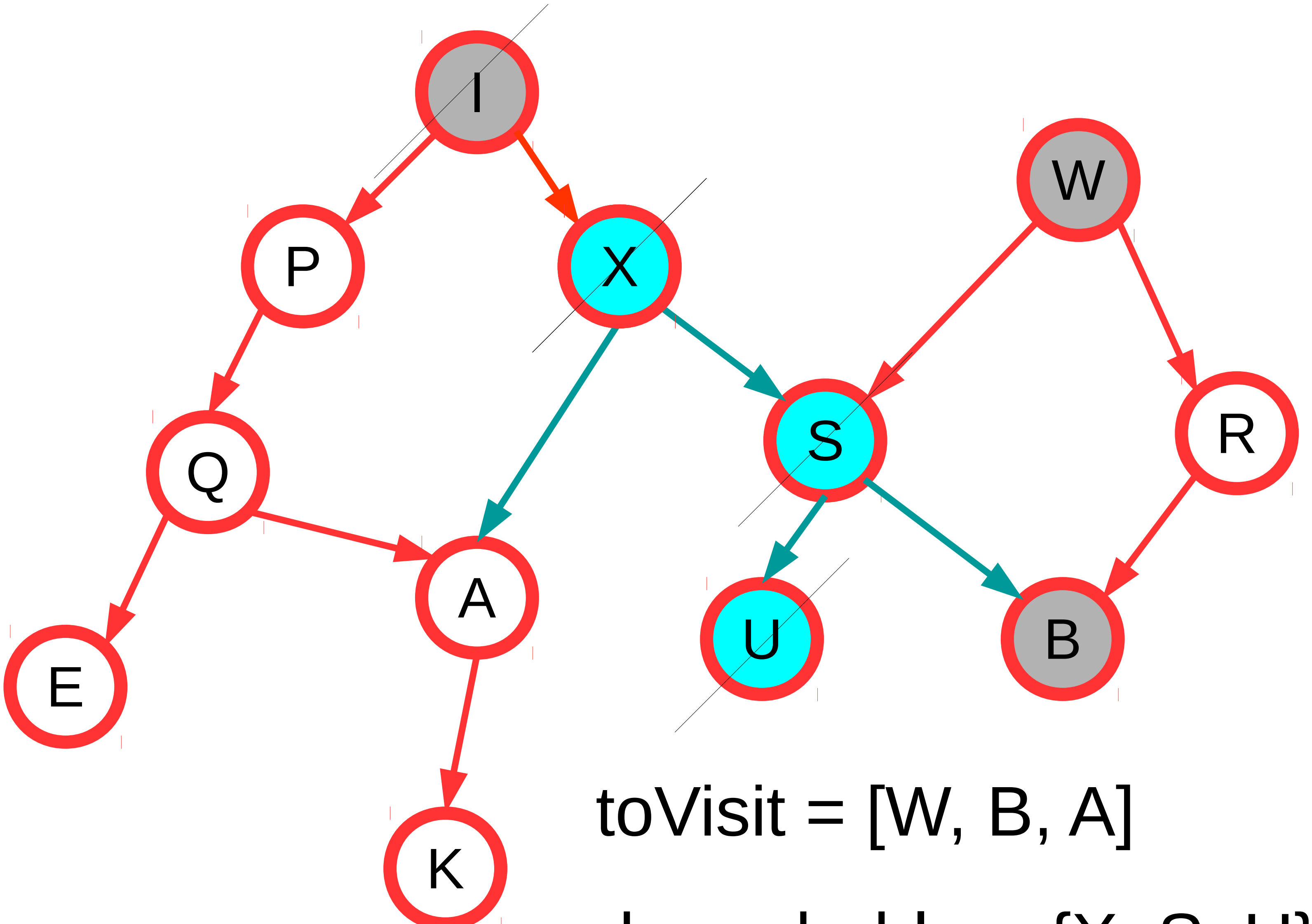
3. Algorithm for d-separation



toVisit = [U, B, W, A]

d-reachable = {X, S}

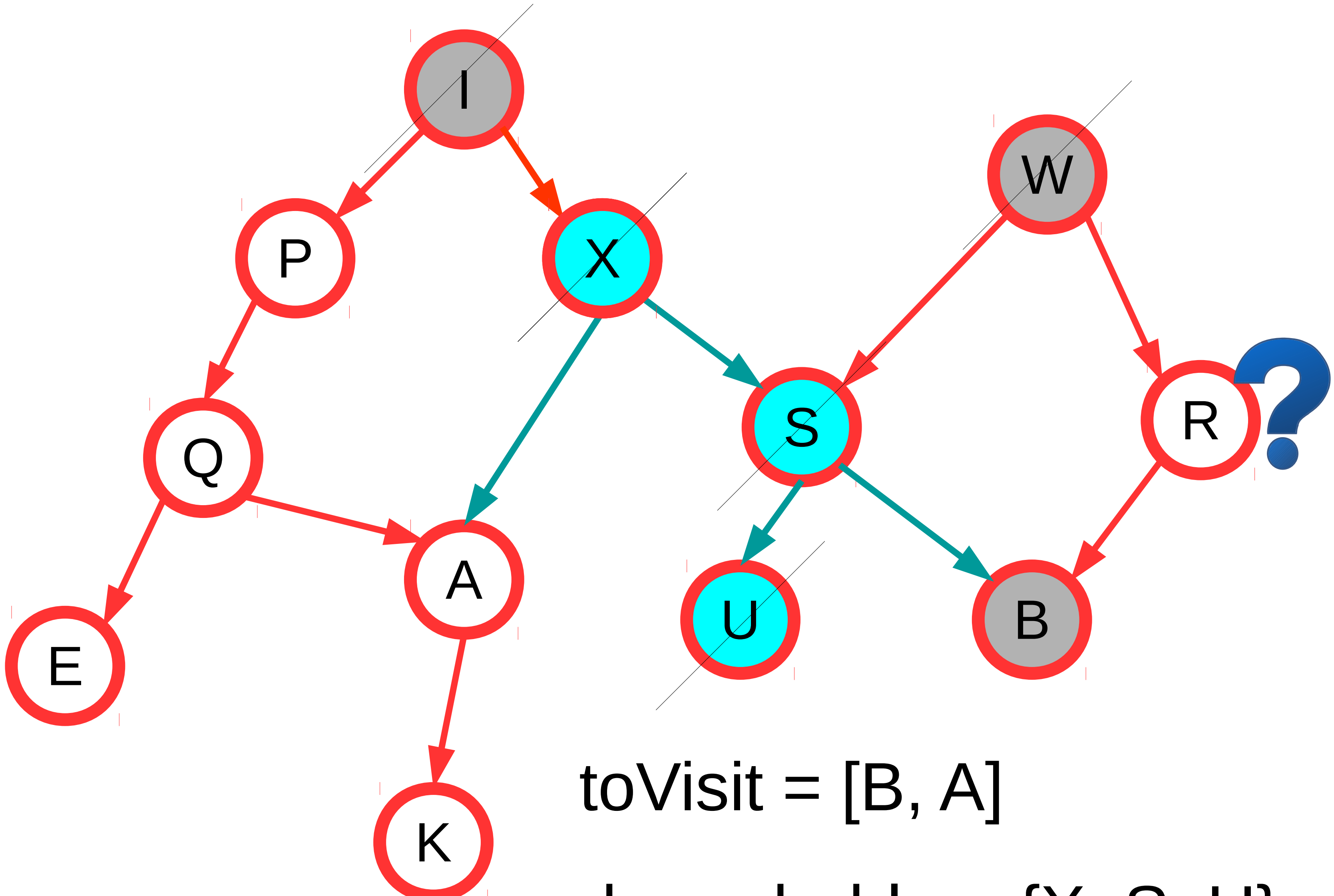
3. Algorithm for d-separation



toVisit = [W, B, A]

d-reachable = {X, S, U}

3. Algorithm for d-separation



toVisit = [B, A]

d-reachable = {X, S, U}

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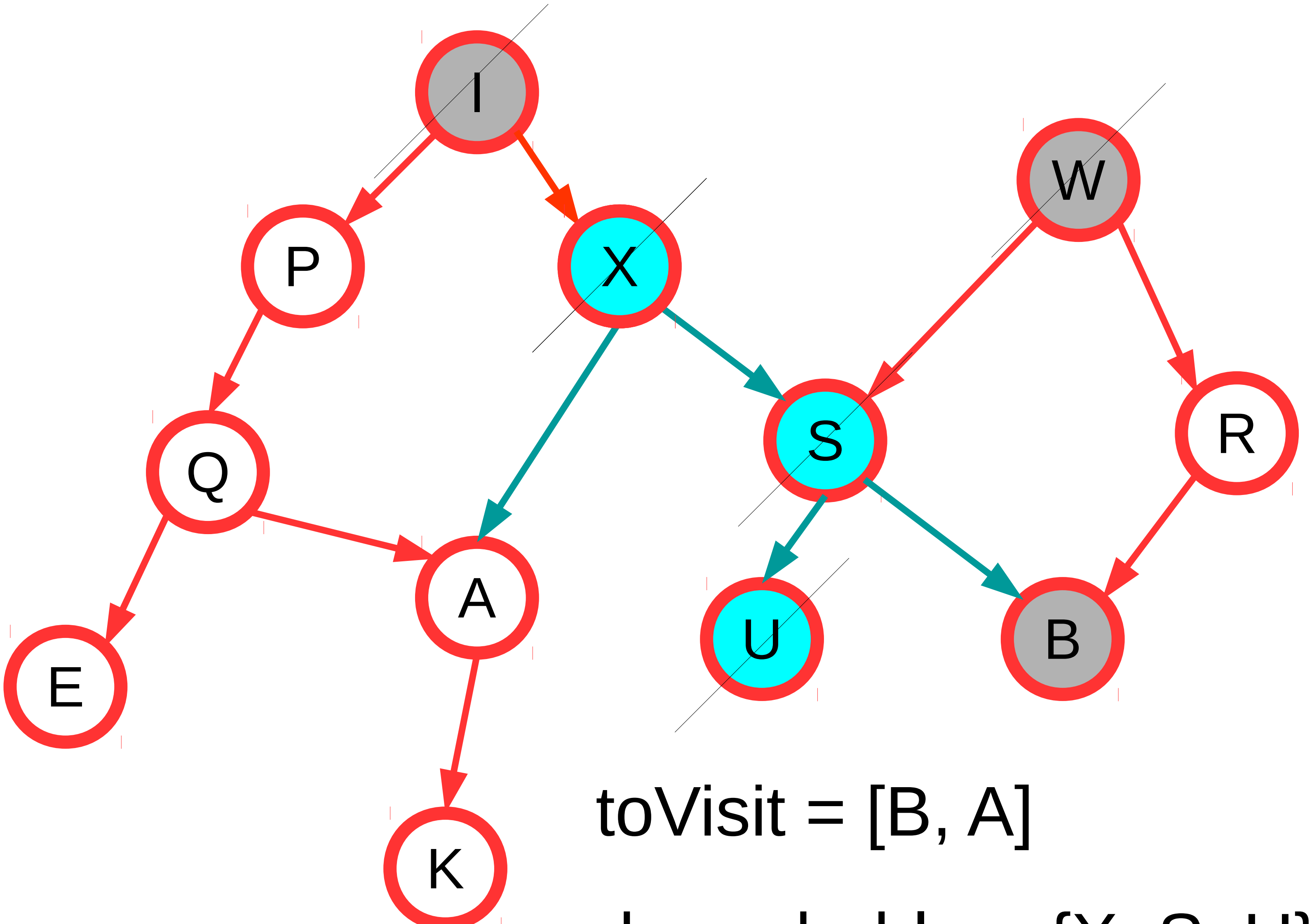
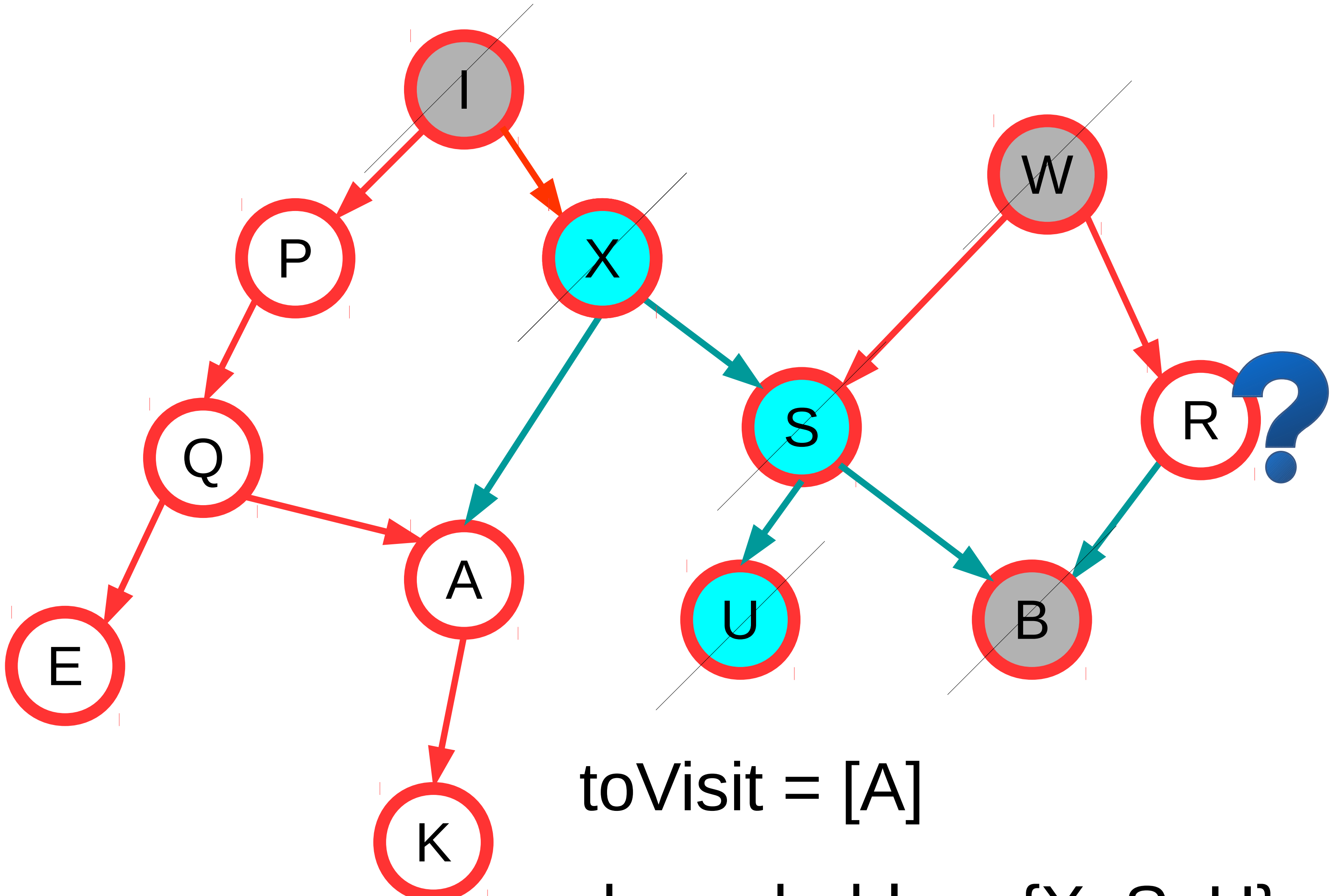


toVisit = [B, A]

d-reachable = {X, S, U}

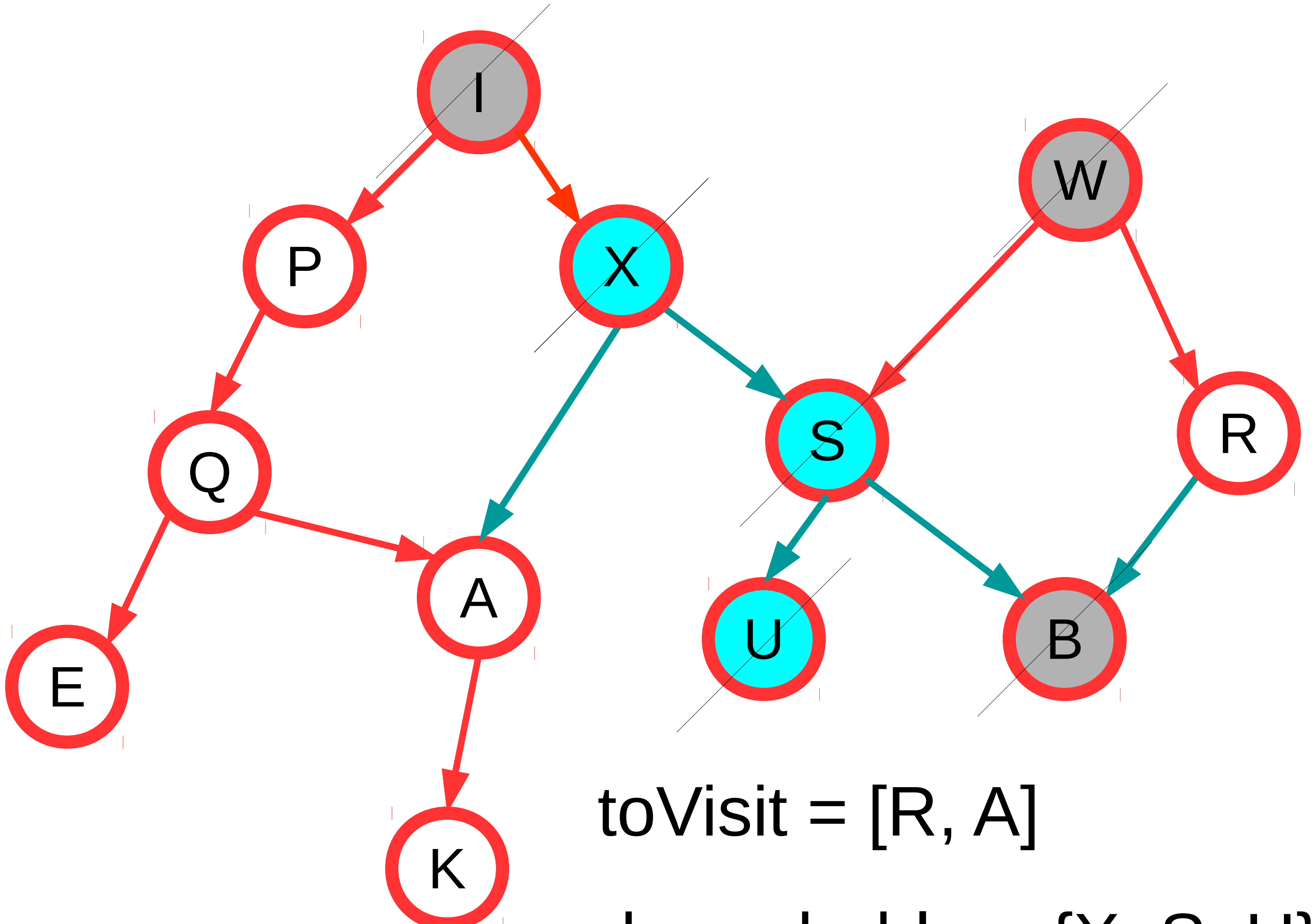
3. Algorithm for d-separation



toVisit = [A]

d-reachable = {X, S, U}

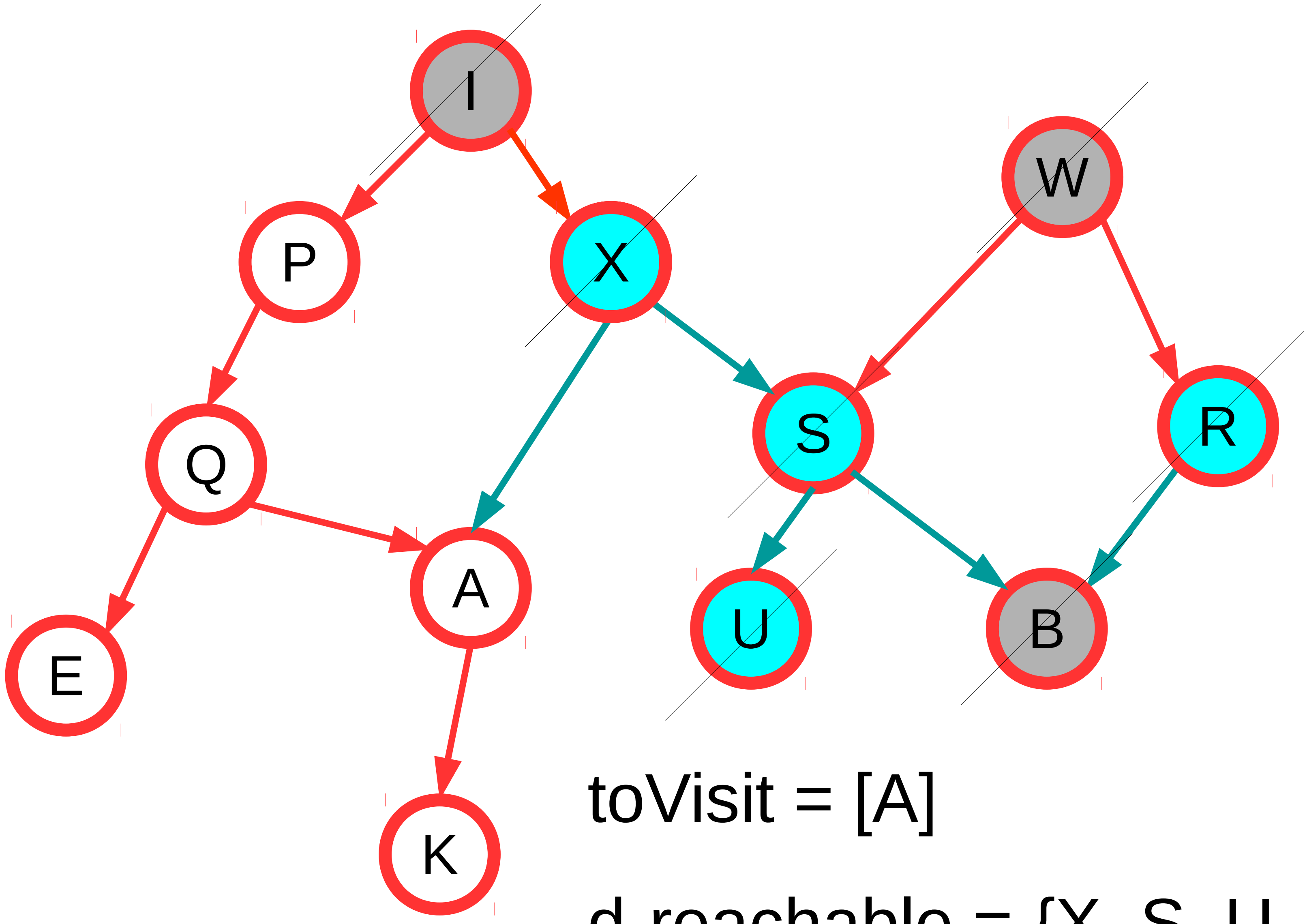
3. Algorithm for d-separation



toVisit = [R, A]

d-reachable = {X, S, U}

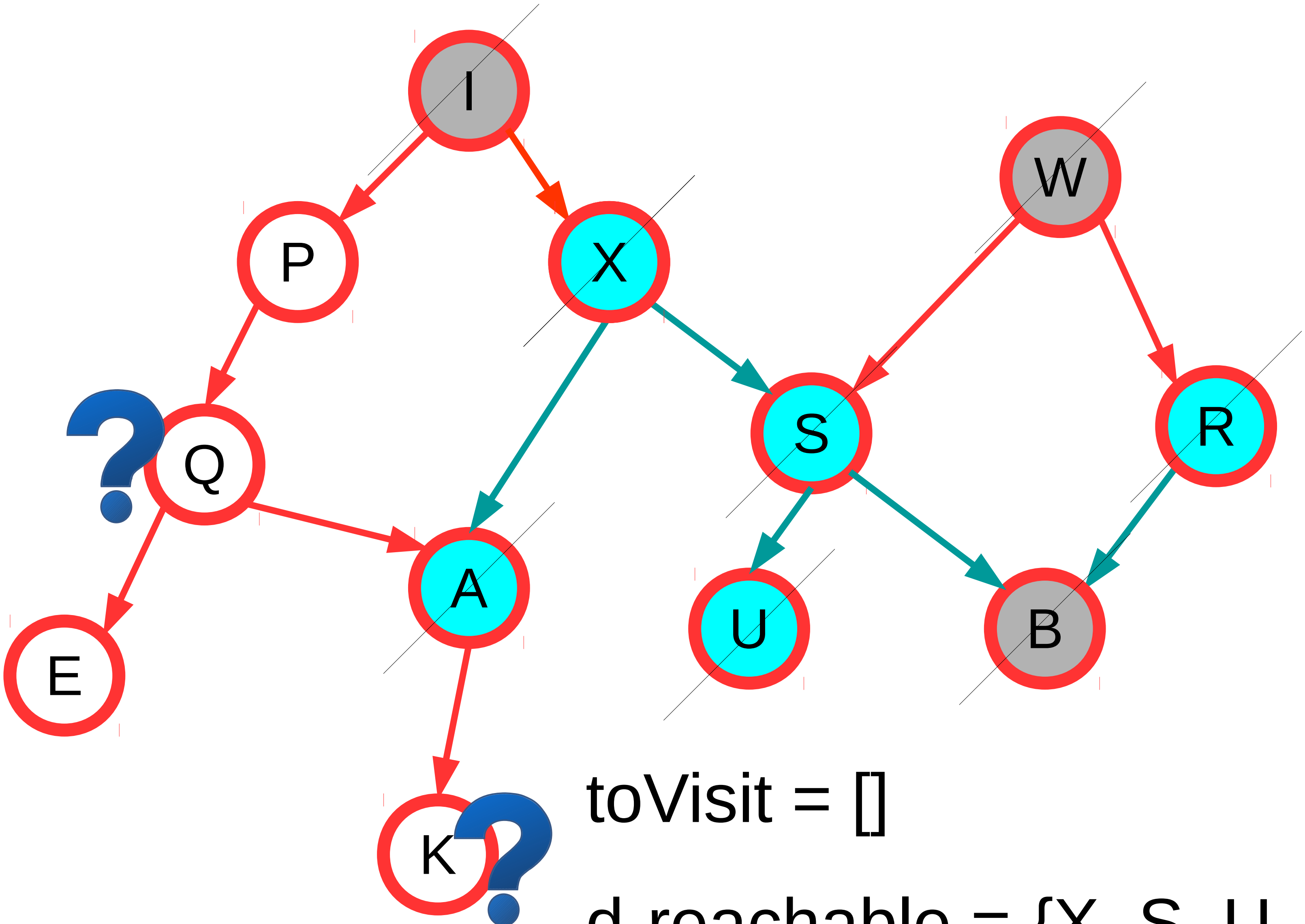
3. Algorithm for d-separation



toVisit = [A]

d-reachable = {X, S, U, R}

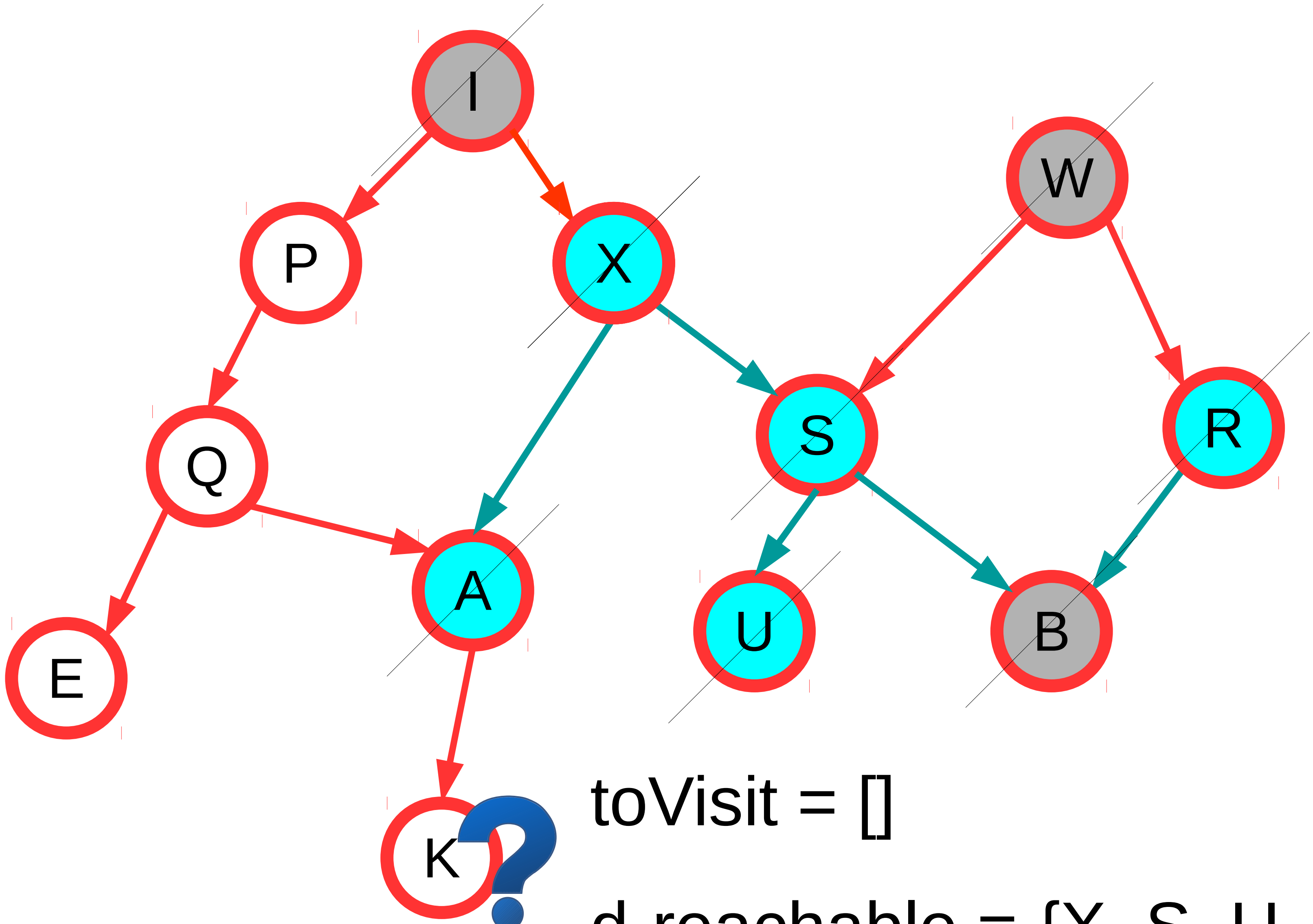
3. Algorithm for d-separation



toVisit = []

d-reachable = {X, S, U, R, A}

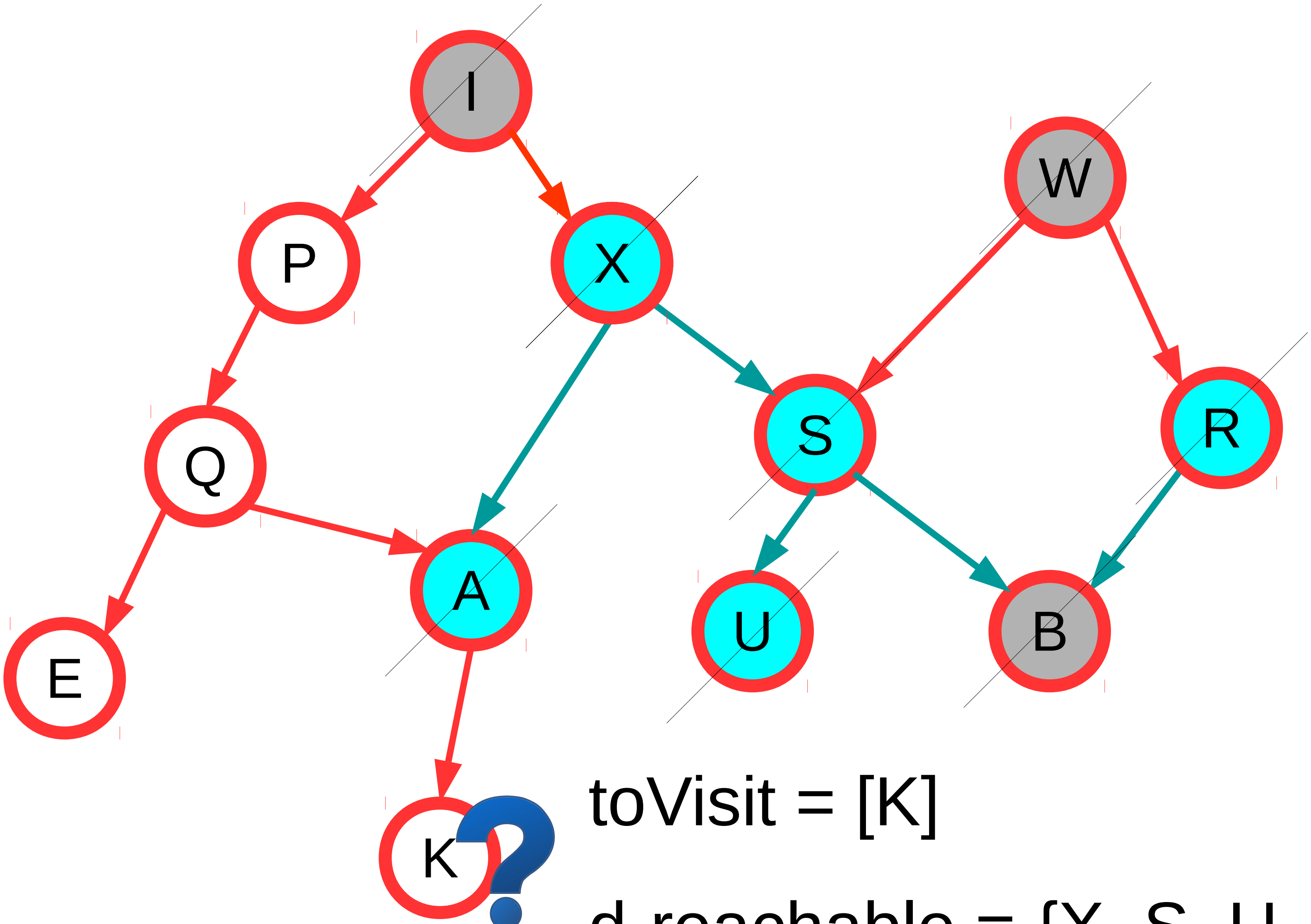
3. Algorithm for d-separation



toVisit = []

d-reachable = {X, S, U, R, A}

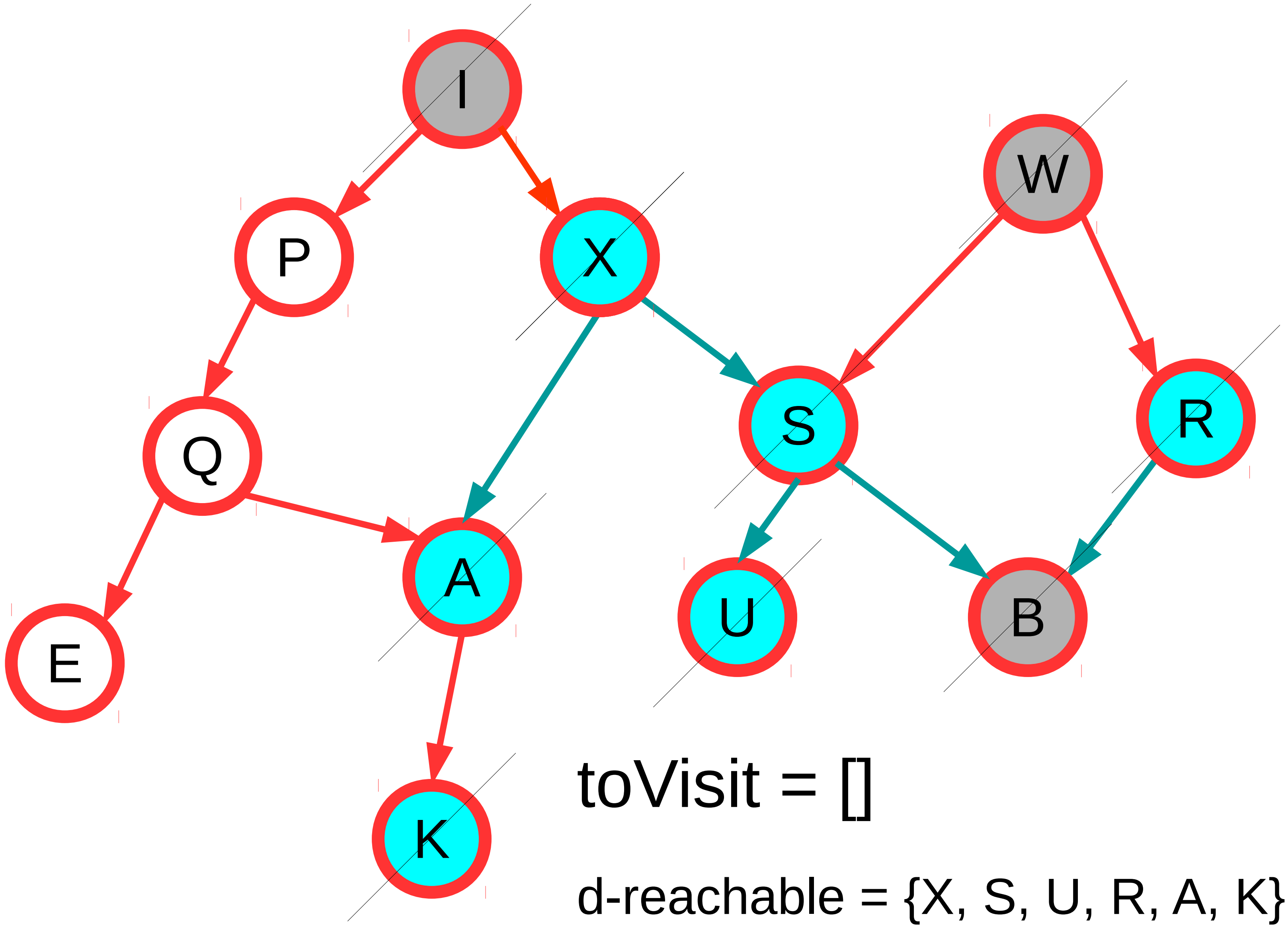
3. Algorithm for d-separation



toVisit = [K]

d-reachable = {X, S, U, R, A}

3. Algorithm for d-separation



toVisit = []

d-reachable = {X, S, U, R, A, K}

3. Algorithm for d-separation

```
def get_reachable(X, E):
    toVisit = [X], visited = {}
    while toVisit != []:
        V = toVisit.pop()
        "visit V"
        add V to visited
        for Y an unvisited neighbor of V:
            if …. then push Y
```

Loop's invariant: Z is in toVisit iff there is an active trail from X to Z

3. Algorithm for d-separation

```
def get_reachable(X, E):
    toVisit = [X], visited = {}, reachable = {}
    while toVisit != []:
        V = toVisit.pop()
        if V is not obs then add V to reachable
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        for Y an unvisited neighbor of V:
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```

Loop's invariant: Z is in toVisit iff there is an active trail from X to Z

What neighbors to append?

- Let V be the node currently visited and Y be an unvisited neighbor.
- By our invariant, there is an active trail **t** from X to V.
- What we need to decide if **t.**append(Y) active...

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    Keep track how you reached V

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def get_reachable(X, E):
    toVisit = [X], visited = {}, reachable = {}
    while toVisit != []:
        V = toVisit.pop()
        if V is not obs then add V to reachable
        add V to visited
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Loop’s invariant: Z is in toVisit iff there is an active trail from X to Z

3. Algorithm for d-separation

```
def get_reachable(X, E):
    toVisit = [X], visited = {}, reachable = {}
    ancestors_E = computeAncestors(E)
    while toVisit != []:
        V = toVisit.pop()
        if V is not obs then add V to reachable
        add V to visited
        for Y an unvisited neighbor of V:
            if …. then push Y
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Loop's invariant: Z is in toVisit iff there is an active trail from X to Z

3. Algorithm for d-separation

def get_reachable(X, **E**):

toVisit = [**(<--,X)**], visited = {}, reachable = {}

ancestors_E = computeAncestors(E)

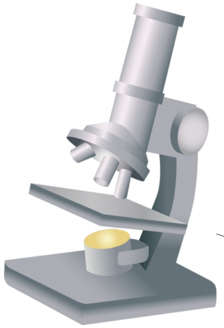
while toVisit != []:

(dir, V) = toVisit.pop()

if V is not obs then add V to reachable

add V to visited

for Y an unvisited neighbor of V:



if …. **then** push Y

Loop's invariant: Z is in toVisit iff there is an active trail from X to Z

for Y an unvisited neighbor of V:

- If dir == <-- :
- If dir == --> :

for Y an unvisited neighbor of V:

- If dir == <-- :
  -
- If dir == --> :
  - If V --> Y:
    -
  - If V <-- Y:
    -

for Y an unvisited neighbor of V:

- If dir == <-- :
  - If V is not observed, then push (dir’, Y)
- If dir == --> :
  - If V --> Y:
    - If V is not observed, then push (-->, Y)
  - If V <-- Y:
    - If V is in ancestors_E, then push (<--, Y)