Efficient Minimization of Decomposable Submodular Functions
By Peter Stobbe (speaking) and Andreas Krause

KEY PROPERTIES FOR MINIMIZATION

Continuous optimization: **convexity**

Discrete optimization: **submodularity**

Real-valued functions of sets
*eg. Classifying pixels, clustering data points, many more*

\[ f(A \cup B) + f(A \cap B) \leq f(A) + f(B) \]

General submodular minimization: \( O^*(n^5) \)

Some restrictive classes (pairwise interactions): efficient
NEW CLASS OF SUBMODULAR FUNCTIONS!  

DECOMPOSABLE

\[ f(A) = \sum_j \phi_j \left( \sum_{k \in A} w_j[k] \right) \]

\( \phi_j \) concave

\( w_j \geq 0 \)

**Example:** Sum of higher order potentials for MAP inference of Markov Random Field

\[ f(A) = \sum_j \left| R_j \setminus A \right| \left| R_j \cap A \right| \]
OVERVIEW OF
Smoothed Lovász Gradient (SLG)

1. Formulate as a convex optimization problem
   \textit{(Lovász 1980)}

2. Apply modern techniques for general nonsmooth convex optimization
   \textit{(Nesterov 2004)}
   Decomposition makes it possible!

3. Use a novel stopping criterion to finish early with optimal answer for discrete problem
Let's see some results!

Matches or outperforms other general-purpose algorithms on standard test problems.

Original

No potentials

Pairwise (Ising) potentials

Higher order potentials (Made possible by SLG)

Matches or outperforms other general-purpose algorithms on standard test problems.

Can solve problems with 10,000 variables in a minute.